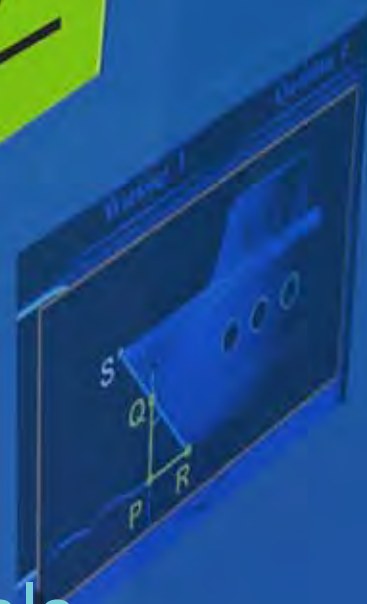
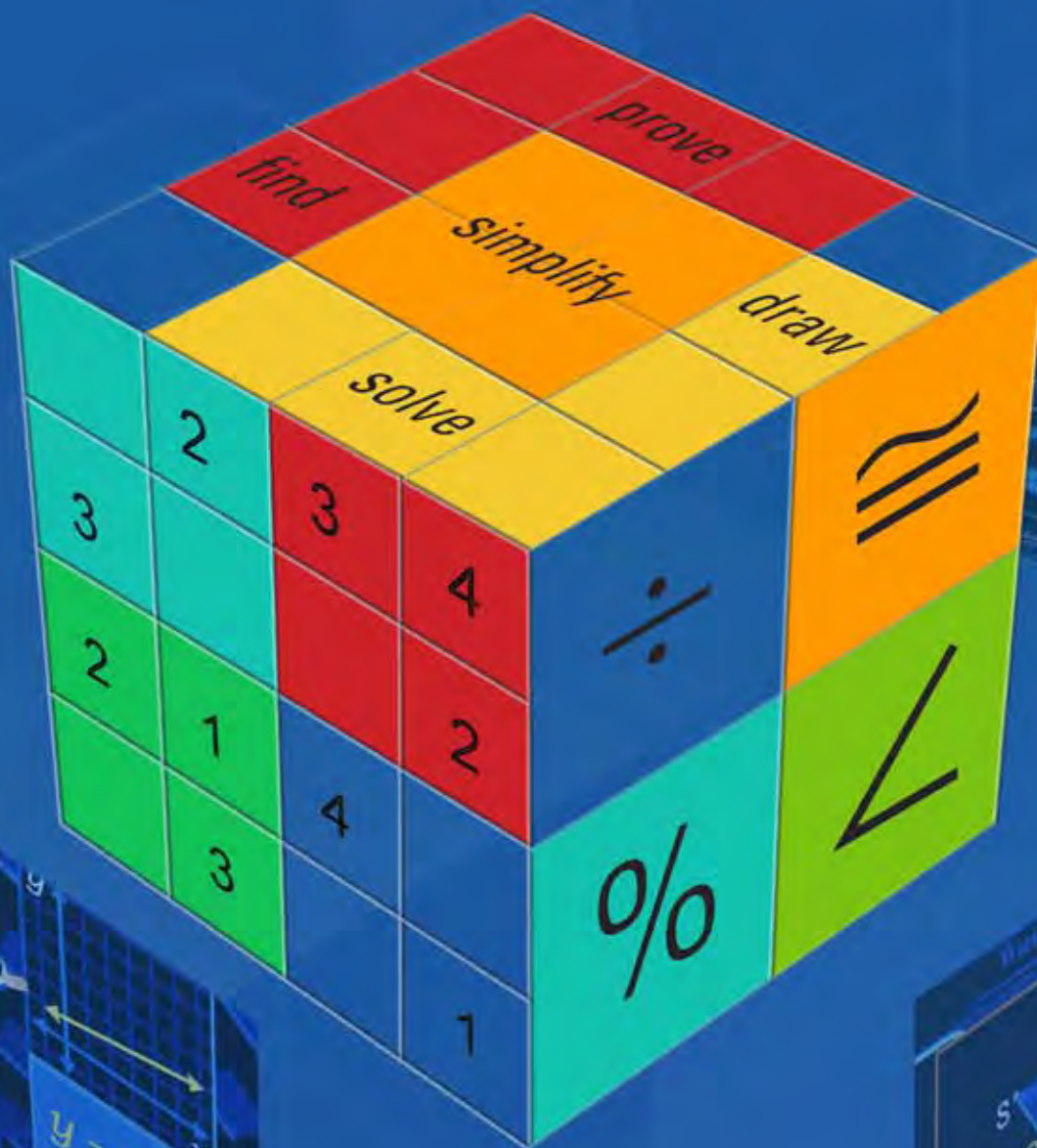




Destination Math California Intervention



Volume 2 Fractions, Decimals
and Negative Integers

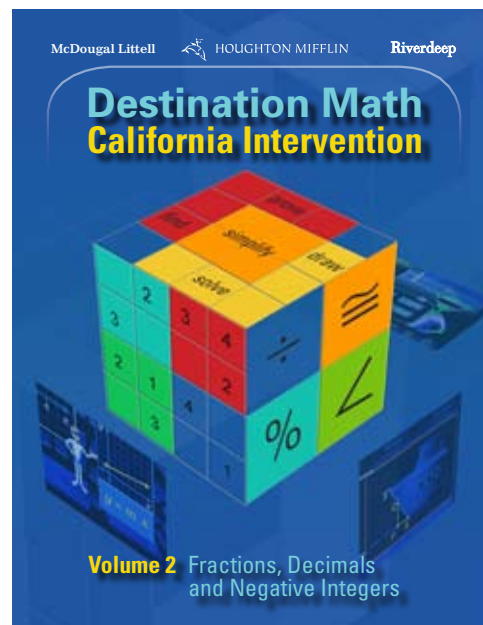
McDougal Littell



California

Destination Math California Intervention

Volume 2: Fractions, Decimals and Negative Integers



McDougal Littell

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Destination Math California Intervention

MATH INTERVENTION

Destination Math California Intervention is a mathematics program designed to help you be successful in future mathematics courses in middle school and high school. Mathematics lessons and problems help you develop mathematical reasoning, understand mathematical terms, and solve multi-step problems.

There are six volumes in DMCI, and four online courses:

Student Volumes

- Volume 1: Fundamental Operations
- Volume 2: Fractions, Decimals and Negative Integers
- Volume 3: Ratios, Rates and Percents
- Volume 4: Properties and Expressions
- Volume 5: Functions, Equations and Data Analysis
- Volume 6: Measurement and Geometry

Mastering Skills and Concepts: Course II Primary Mathematics

Mastering Skills and Concepts: Course III Basic Arithmetic

Mastering Skills and Concepts: Course IV Basic Mathematics

Mastering Skills and Concepts: Course V Pre Algebra

In the six volumes, lessons include examples of problems that are worked out, and Try This exercises to help you solve similar problems in The Practice section. In Did You Get It? you show your understanding of the concepts, procedures and vocabulary. Problem solving lessons teach strategies for approaching problems and how to check whether or not your answers make sense. Activities are also included to help you understand new concepts using models and games. After each group of lessons, you practice vocabulary, skills, and problem-solving skills in a section called Mixed Practice.

Continued...

MATH INTERVENTION cont.

In the four online courses, you will meet interesting characters who explain math concepts as they solve problems. Interactive Tutorials take you through a math concept, step-by-step. You'll answer questions and solve problems as you go. Characters in the program give you hints and explain things if you don't select the correct answer. You work at your own pace, and use the Explore option for more Tutorials or to help you remember previous lessons. Remember to take notes as you go through the Tutorial.

After the Tutorial, you will apply what you learned by solving a Workout question. The colors in the workout buttons show you the level of difficulty of the problems: blue (easiest), green (medium), purple (most challenging). Your teacher may assign additional practice problems called Your Turn and Unit Review, as well as tests.

Your teacher might assign you to complete a whole volume, or selected lessons in a volume, and one or more lessons, units or modules in the online course.

LESSON
2-1

California
Standards

Gr. 4 NS 4.1: Understand that many whole numbers break down in different ways (e.g., $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$).

Gr. 5 NS 1.4: Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

Prime Factorization

Words to Remember

Factor: When two or more numbers are multiplied together, they are called factors.

Factors

$$\begin{array}{c} \downarrow \quad \downarrow \\ 2 \times 3 = 6 \end{array}$$

Prime number: A whole number greater than zero that has only itself and 1 as factors

Prime number

$$\begin{array}{c} \downarrow \\ 1 \times 5 = 5 \end{array}$$

Composite number: A whole number greater than 1 that has factors other than itself and 1

Composite number

$$\begin{array}{c} \downarrow \qquad \qquad \downarrow \\ 1 \times 6 = 6 \text{ or } 2 \times 3 = 6 \end{array}$$

Getting Started You learned how to find the product of two factors. You can also write numbers as the product of their factors.

EXAMPLE 1 Finding the Factors of a Number

Find all the factors of 24.

Solution

Step 1 Write 24 as a product of two factors in as many ways as possible.

Factors of 24

$$1 \times 24$$

$$2 \times 12$$

$$3 \times 8$$

$$4 \times 6$$

$$\text{Stop.} \quad \begin{array}{c} \nearrow \\ 6 \times 4 \end{array}$$

Start with 1 and continue with the next highest factor until a pair of factors repeats.

Repeated Factors

When a factor repeats, you only list it once. For example, you can write 25 as 1×25 or as 5×5 , but you say that the factors of 25 are 1, 5, and 25.

Step 2 List the factors in order from least to greatest.

ANSWER The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

TRY THIS Find all the factors of the number.

- The factors of 18 are , , , , , and .
- The factors of 35 are , , , and .

EXAMPLE 2**Identifying Prime and Composite Numbers**

Tell whether the number is *prime* or *composite*.

a. 15

b. 11

Solution

a. The factors of 15 are 1, 3, 5, and 15, so 15 is composite.

b. The factors of 11 are 1 and 11, so 11 is prime.

TRY THIS

Tell whether the number is *prime* or *composite*.

3. 43 _____

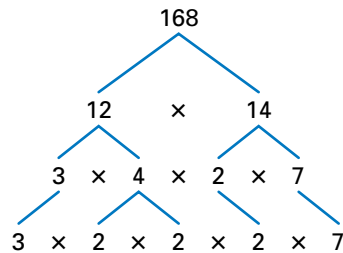
4. 27 _____

Prime Factorization Writing the prime factorization of a number means writing a number as the product of prime numbers. You can use a factor tree to write the prime factorization of a number.

EXAMPLE 3**Writing the Prime Factorization of a Number**

Write the prime factorization of 168.

Solution



Write the given number as a product of any two factors.

Continue to write each factor as a product of two factors until all factors are prime numbers.

The prime number factors of 168 are 2^3 , 3, and 7. The factor 2 has an exponent because it is used 3 times.

ANSWER The prime factorization of 168 is $2^3 \times 3 \times 7$.

TRY THIS

Write the prime factorization of the number.

5. 200 $\square \times \square \times \square \times \square \times \square = \square \times \square$

6. 15 $\square \times \square$

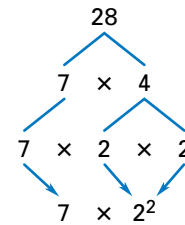
Summarize**Finding the Factors of a Number**

Write the number as a product of two factors in as many ways as possible. List the factors from least to greatest.

Writing the Prime Factorization of a Number

Write the number as a product of any two factors. Continue to write each factor as a product of two factors until all factors are prime.

$$40 \rightarrow 1, 2, 4, 5, 8, 10, 20, 40$$

**Practice**

Find all the factors of the number.

- | | | | |
|-------|---------|---------|---------|
| 1. 2 | 2. 8 | 3. 17 | 4. 25 |
| 5. 36 | 6. 49 | 7. 60 | 8. 73 |
| 9. 99 | 10. 108 | 11. 121 | 12. 200 |

Tell whether the number is *prime* or *composite*.

- | | | | |
|--------|--------|---------|---------|
| 13. 9 | 14. 11 | 15. 21 | 16. 33 |
| 17. 47 | 18. 52 | 19. 65 | 20. 76 |
| 21. 91 | 22. 95 | 23. 100 | 24. 148 |

Match the number with its prime factorization.

- | | |
|--------------|------------------|
| 25. 80 _____ | A. $2 \cdot 3^3$ |
| 26. 18 _____ | B. $2^3 \cdot 3$ |
| 27. 24 _____ | C. $2^4 \cdot 5$ |
| 28. 54 _____ | D. $2 \cdot 3^2$ |

Write the prime factorization of the number.

- | | | | |
|---------|---------|---------|---------|
| 29. 7 | 30. 12 | 31. 20 | 32. 36 |
| 33. 43 | 34. 58 | 35. 64 | 36. 92 |
| 37. 125 | 38. 134 | 39. 157 | 40. 170 |
| 41. 220 | 42. 325 | 43. 344 | 44. 561 |

First tell whether you will *find factors*, *identify prime and composite numbers*, or *write a prime factorization* to find the answer. Then solve the problem. Explain your reasoning.

- 45.** You are dividing **30** tomato plants into equal rows for planting. How many possibilities are there?

- 46.** Since you live at **240** Wilkens Lane, you decide to write **240** as a product of prime numbers and use each factor as a digit in a numeric password. How many digits will you have? Explain.



- 47.** You are using the number from a family member's birthday as the number on the back of your football jersey. You want the number to be divisible only by itself and **1**. Your family members were born on February **16**, June **13**, August **12**, and August **27**. Which birthday will you use?

DID YOU GET IT?

- 48. Fill in the missing words.** To write the prime factorization of a number, first write the number as a product of two _____.
- 49. Explain your reasoning.** Your friend says the factors of **90** are all prime. Is your friend correct? Explain why or why not.

LESSON
2-2

California Standards

Gr. 6 NS 2.4: Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Greatest Common Factor

Words to Remember

Common factor: Any whole number that is a factor of two or more non-zero whole numbers

Greatest common factor (GCF): The largest whole number that is a factor of two or more non-zero whole numbers

Common factors of 8 and 12



Factors of 8: **1, 2, 4, 8**

Factors of 12: **1, 2, 3, 4, 6, 12**

Greatest common factor of 8 and 12



Factors of 8: 1, 2, **4**, 8

Factors of 12: 1, 2, 3, **4**, 6, 12

Getting Started In Lesson 2-1 you learned how to find the factors of a number. You can also use the factors of two or more numbers to find the greatest common factor (GCF).

EXAMPLE 1

Finding Common Factors

Find all the common factors of 20 and 30.

Solution

Step 1 List all the factors of 20 and 30.

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Step 2 Circle the factors that appear in both lists.

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

ANSWER The common factors of 20 and 30 are 1, 2, 5, and 10.

TRY THIS

Find all the common factors of the numbers.

1. 14 and 34 Factors of 14: , , 7,

_____ Factors of 34: , , , 34

2. 28 and 72 Factors of 28: 1, 2, , , ,

_____ Factors of 72: , , , , 6, , , 12, , , ,

Repeated Factors

When a factor repeats, you only list it once. For example, for the factors of 36, only list the 6 once.

EXAMPLE 2**Finding the GCF Using a List**

Find the greatest common factor of 36 and 48.

Solution

Step 1 List all the factors of 36 and 48.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Step 2 Identify and circle the greatest factor that appears in both lists.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

ANSWER The GCF of 36 and 48 is 12.

TRY THIS

Make a list to find the GCF.

3. 16 and 24

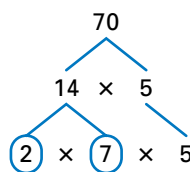
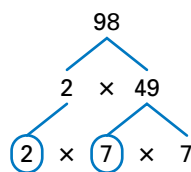
4. 9 and 27

EXAMPLE 3**Finding the GCF Using Prime Factorization**

Find the greatest common factor of 98 and 70.

Solution

Write the prime factorizations of 98 and 70.



Circle all the common prime factors.

The common prime factors are 2 and 7. The GCF is the product of the common prime factors.

ANSWER The GCF of 98 and 70 is 2×7 , or 14.

TRY THIS

Use prime factorization to find the GCF.

5. 52 and 96

6. 126 and 210



Summarize**Finding the GCF Using a List**

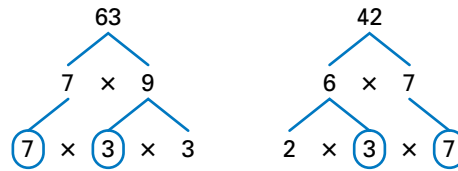
List all the factors of the numbers. Identify the greatest factor that appears in each list.

63: 1, 3, 7, 9, **21**, 63 GCF is 21.

42: 1, 2, 3, 6, 7, 14, **21**, 42

Finding the GCF Using Prime Factorization

Write the prime factorization of the numbers.
Identify all the common prime factors.
Multiply them together to find the GCF.



GCF is $7 \times 3 = 21$.

Practice

Fill in the missing information to find all the common factors of the numbers.

1. 6 and 8

Factors of 6: , , ,

Factors of 8: , , ,

2. 9 and 18

Factors of 9: , ,

Factors of 18: , , , , ,

3. 24 and 38

4. 30 and 75

Make a list to find the greatest common factor of the numbers.

5. 4 and 74

Factors of 4: , ,

Factors of 74: , , ,

GCF: _____

6. 21 and 49

Factors of 21: , , ,

Factors of 49: , ,

GCF: _____

7. 56 and 64

GCF: _____

8. 15 and 105

GCF: _____

Use prime factorization to find the greatest common factor of the numbers.

9. 16 and 72



GCF: _____

10. 60 and 90



GCF: _____

11. 45 and 120

GCF: _____

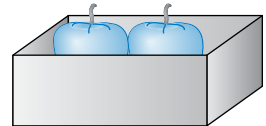
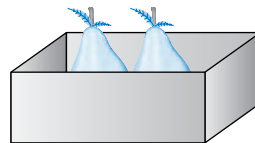
12. 100 and 180

GCF: _____

First tell whether you will find *common factors* or the *greatest common factor*. Then solve the problem. Explain your reasoning.

- 13.** Ms. Randolph wants to divide Mr. Hunt's class of **14** students and Ms. Cary's class of **21** students into smaller groups of equal size. She does not want to mix the classes. What is the greatest number of students that she can put in one group?

- 14.** You have **30** apples and **40** pears. You are dividing the fruit into equal groups without mixing types of fruit. Name all possible group sizes. The groups must have more than one piece of fruit.



- 15.** You have **220** black-and-white photos and **400** color photos to display on posters. You want to put the same number of photos on each poster without mixing black-and-white photos with color photos. What is the greatest number of photos you can put on each poster?

DID YOU GET IT?

- 16. Fill in the missing words.** To find the GCF using prime factorization, determine the _____ of all the common prime factors.

- 17. Work backwards.** The GCF of two numbers is **6**. What could the numbers be? Explain.

LESSON
2-3

California
Standards

Gr. 6 NS 2.4: Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Least Common Multiple

Words to Remember

Multiple: Any whole number that is a product of the number and another whole number

Common multiple: A multiple shared by two or more numbers

Least common multiple (LCM): The smallest multiple shared by two or more numbers

Multiples of 2

$$2 \times 1 = 2$$

$$2 \times 2 = 4$$

$$2 \times 3 = 6$$

Common multiples of 6 and 8

Multiples of 6: 6, 12, 18, **24**, 30, 36, 42, **48**, ...

Multiples of 8: 8, 16, **24**, 32, 40, **48**, 56, 64, ...

Least common multiple of 6 and 8

Multiples of 6: 6, 12, 18, **24**, 30, 36, 42, 48, ...

Multiples of 8: 8, 16, **24**, 32, 40, 48, 56, 64, ...

Getting Started You learned how to find the product of two whole numbers. You can use products to find the least common multiple (LCM).

EXAMPLE 1

Finding Multiples

List the multiples of 9 up through 80.

Solution

Multiply 9 by whole numbers starting with 1. Each new product should be 9 greater than the previous product. List the products until you reach the product closest to, but not greater than, 80.

$$9 \times 1 = 9$$

$$9 \times 2 = 18$$

$$9 \times 3 = 27$$

$$9 \times 4 = 36$$

$$9 \times 5 = 45$$

$$9 \times 6 = 54$$

$$9 \times 7 = 63$$

$$9 \times 8 = 72$$

ANSWER The multiples of 9 up through 80 are 9, 18, 27, 36, 45, 54, 63, and 72.

TRY THIS

List the multiples of the number up through 100.

1. 20

$$20 \times \square = \square$$

$$20 \times \square = \square$$

$$20 \times \square = \square$$

$$20 \times \square = \square$$

$$20 \times \square = \square$$

2. 16

$$16 \times \square = \square$$

$$16 \times \square = \square$$

$$16 \times \square = \square$$

$$16 \times \square = \square$$

$$16 \times \square = \square$$

EXAMPLE 2**Finding the LCM Using a List**

Find the least common multiple of 6 and 14.

Solution

Step 1 Start listing the multiples of 6 and 14.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 14: 14, 28, 42, 56, ...

Step 2 Identify and circle the smallest multiple that appears in both lists.

Multiples of 6: 6, 12, 18, 24, 30, 36, **42**, 48, ...

Multiples of 14: 14, 28, **42**, 56, ...

ANSWER The LCM of 6 and 14 is 42.

TRY THIS

Make a list to find the LCM.

3. 5 and 8

4. 2, 3, and 4

EXAMPLE 3**Finding the LCM Using Prime Factorization**

Find the least common multiple of 42 and 56.

Solution

Step 1 Write the prime factorizations of 42 and 56. $42 = 2 \times 3 \times 7$

Step 2 Circle all the common prime factors. $56 = 2 \times 2 \times 2 \times 7$

Step 3 Multiply all the prime factors together, using each common factor only once.

Common factors Remaining factors

$$2 \times 7 \times 3 \times 2 \times 2 = 168$$

ANSWER The LCM of 42 and 56 is 168.

TRY THIS

Use prime factorization to find the LCM.

5. 32 and 48

6. 12, 120, and 180

Summarize

Finding the LCM Using a List

Start listing multiples of the numbers.
Identify the smallest multiple that appears in each list.

6: 6, 12, 18, 24, **30**, 36, ...

15: 15, **30**, 45, ... LCM: 30

10: 10, 20, **30**, 40, ...

Finding the LCM Using Prime Factorization

Write the prime factorizations of the numbers.
Find the product of all the prime factors.
Use each common factor only once.

$18 = 2 \times \textcircled{3} \times \textcircled{3}$

$27 = \textcircled{3} \times \textcircled{3} \times 3$

LCM: $3 \times 3 \times 2 \times 3 = 54$

Practice

1. Find the multiples of 7 through 75.

7, , , , , , , , ,

2. Find the multiples of 6 through 80.

, , 18, , , , , , , , , ,

Make a list to find the least common multiple of the numbers.

3. 4 and 6

4. 3 and 15

5. 5 and 9

6. 8 and 10

7. 48 and 80

8. 3, 7, and 21

Use prime factorization to find the least common multiple of the numbers.

9. 27 and 36

10. 16 and 72

11. 4, 5, and 10

12. 12, 15, and 24

13. 39 and 52

14. 44 and 77

Solve the problem. Explain your reasoning.

- 15.** Snack pouches come in packs of **8**, and drink boxes come in packs of **12**. There are just enough packs of each item that each snack pouch is paired with one drink. How many of pairs of snack pouches and drink boxes are there?



- 16.** Katy lifts weights every **4** days and swims laps every **6** days. Every **14** days she takes a yoga class. What day of her exercise routine is the first time she is supposed do all three activities?

- 17.** Carson inspects bike helmets at a factory. He notices that every **36th** helmet has a crack in the plastic. Every **48th** helmet is missing a buckle. Which is the first helmet that Carson will inspect that is both cracked and missing a buckle?

DID YOU GET IT?

- 18. Fill in the missing words.** When using a list to find the LCM, you identify the _____ of the common _____ that appears in each list.
- 19. Work backwards.** The prime factorization of a number is $2^2 \times 3^2$. Find another number so that the LCM of both numbers is **180**.

- 19.** Fill in the missing information to solve the problem.

Your grocery store sells hamburger patties in packs of **6** and buns in packs of **8**. You buy just enough packs of each so you can match each patty with a bun. How many hamburger sandwiches can you make?

Step 1 The phrases “buy just enough packs” and “match each patty with a bun” tell you that you need to find the _____.

Step 2 Make two lists: **8**: _____

6: _____

Step 3 You can make _____ hamburger sandwiches.

Find the LCM of the numbers.

20. 5 and 12

21. 2 and 6

22. 14 and 10

23. 16 and 20

24. 56 and 8

25. 9 and 15

26. 3 and 21

27. 35 and 25

28. 42 and 7

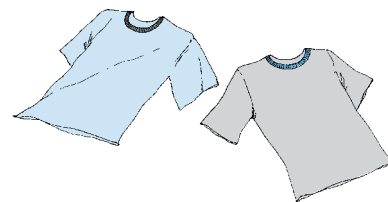
29. 11 and 2

30. 18 and 8

31. 40 and 85

First tell whether you need to find the *greatest common factor* or the *least common multiple* to find the answer. Then solve the problem. Explain your reasoning.

- 32.** A T-shirt shop has **50** blue shirts and **20** gray shirts. An employee separates the shirts into equal-sized groups without mixing the colors. How many groups are there?



- 33.** The accountant at a travel agency pays the water bill every **3** months and the insurance bill every **4** months. If both bills are paid together this month, in how many months will both bills be paid together again?

LESSON 2-4



California Standards

Gr. 4 NS 1.5:

Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions (see Standard 4.0).

Gr. 4 NS 1.7: Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.

Also included: Gr. 2 NS 4.0, Gr. 2 NS 4.1, Gr. 4 NS 1.6

Introduction to Fractions

Words to Remember

Fraction: A number written in the form $\frac{a}{b}$ that refers to parts of a set or parts of a whole

Fraction: $\frac{4}{5}$ ← fraction bar

Numerator: The number above the fraction bar that refers to the equal parts out of a set or whole

Denominator: The number below the fraction bar that refers to a whole or to the total number of equal parts in a set

$\frac{4}{5}$ ← Numerator
← Denominator

Getting Started You learned how to read, write, and represent whole numbers. You can also read, write, and represent fractions.

EXAMPLE 1 Reading and Writing Fractions

- a. Write the fraction $\frac{4}{9}$ in words. b. Write six sevenths as a fraction.

Solution

- a. Write the numerator followed by the appropriate name for the denominator.

$$\frac{4}{9} \rightarrow \frac{\text{four}}{\text{ninths}}$$

ANSWER four ninths

- b. Write the first number, the numerator, over a fraction bar. Then write the second number, the denominator, under the fraction bar.

$$\text{six sevenths} \rightarrow \frac{\text{six}}{\text{sevenths}}$$

ANSWER $\frac{6}{7}$

TRY THIS Write the fraction in words or the words as a fraction.

1. $\frac{3}{5}$

2. one twelfth

Reading Fractions

Denominators of fractions are named as follows:

2 → half

3 → third

4 → fourth

5 → fifth

6 → sixth

7 → seventh

8 → eighth

9 → ninth

10 → tenth

11 → eleventh

12 → twelfth

100 → hundredth

and so on ...

If the numerator is 1, use the form given above. If the numerator is greater than 1, add an s to make the denominator plural. The plural of *half* is *halves*.

EXAMPLE 2**Representing a Fraction Using a Model****Parts of a Whole**

The model drawn in Example 2 shows how fractions can represent parts of a whole.

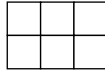
Draw a model that represents $\frac{5}{6}$.

Solution

Step 1 Draw a rectangle that is divided into six equal parts.

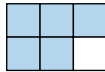
The denominator tells the total number of equal parts.

The denominator of $\frac{5}{6}$ is 6.



Step 2 Shade five of the six parts. The numerator tells how many parts of the whole. The numerator of $\frac{5}{6}$ is 5.

ANSWER

**Parts of a Set**

The model in Example 3 illustrates how fractions can represent parts of a set.

TRY THIS

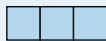
Draw a model that represents the fraction.

3. $\frac{7}{10}$

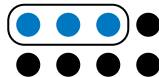
4. $\frac{2}{3}$

Using a Model

$\frac{3}{3}$ means
3 parts out of 3 which
equals 1

**EXAMPLE 3****Writing a Fraction Represented by a Model**

Write the fraction that is represented by the model.

**Solution**

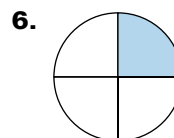
Count the number of circled beads. That number is the numerator. Then count the total number of beads. That number is the denominator.

$$\frac{\text{number of circled beads}}{\text{total number of beads}} = \frac{3}{8}$$

ANSWER $\frac{3}{8}$

TRY THIS

Write the fraction that is represented by the model.



Summarize**Reading and Writing Fractions**

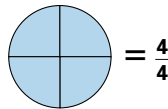
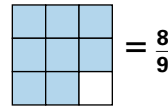
Write the numerator over the denominator.

Using Models

The shaded parts in a model represent the numerator of a fraction. The total number of parts in a model represent the denominator of a fraction.

The fraction "one half" is written as

$$\frac{1}{2} \quad \begin{array}{l} \leftarrow \text{Numerator} \\ \leftarrow \text{Denominator} \end{array}$$

**Practice**

Write the fraction in words.

1. $\frac{5}{8}$ _____

2. $\frac{4}{5}$ _____

3. $\frac{9}{10}$ _____

4. $\frac{3}{7}$ _____

5. $\frac{8}{11}$ _____

6. $\frac{5}{6}$ _____

7. $\frac{16}{100}$ _____

8. $\frac{9}{12}$ _____

9. $\frac{7}{9}$ _____

Write the words as a fraction.

10. two thirds



11. three fourths



12. one sixth



13. four sevenths



14. two ninths



15. six elevenths



Draw a model that represents the fraction.

16. $\frac{1}{2}$

17. $\frac{3}{4}$

18. $\frac{4}{11}$

19. $\frac{7}{8}$

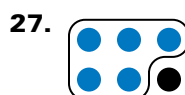
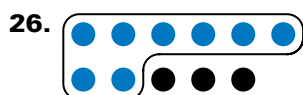
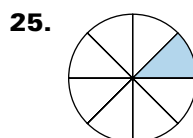
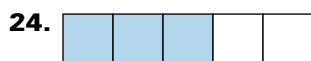
20. $\frac{1}{3}$

21. $\frac{2}{9}$

22. $\frac{5}{7}$

23. $\frac{2}{5}$

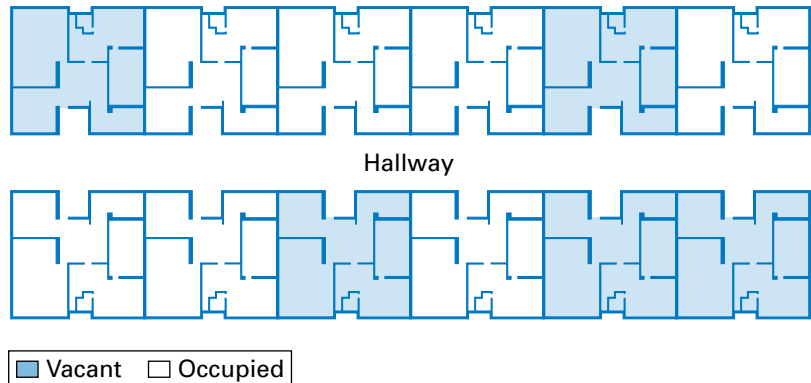
Write the fraction that is represented by the model.



First identify the *numerator* and *denominator*. Then write a fraction to represent the situation. Explain your reasoning.

28. Three students in a class of eleven sign up for student council.

29. The diagram shows the number of vacant apartments in one wing of an apartment complex.

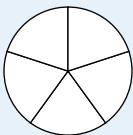


30. Kim exercises four afternoons in seven days.

DID YOU GET IT?

31. **Fill in the missing words.** The _____ of a fraction describes the equal parts of a whole. The _____ of a fraction describes the total number of parts.

32. **Use a model.** There is $\frac{2}{5}$ of a chicken pot pie left over from dinner last night. Fill in the blank model to represent this situation.



LESSON
2-5

California
Standards

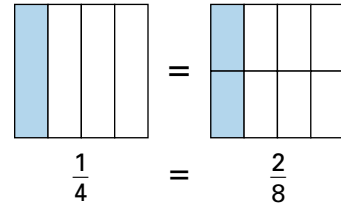
Gr. 3 NS 3.1: Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $\frac{1}{2}$ of a pizza is the same amount as $\frac{2}{4}$ of another pizza that is the same size; show that $\frac{3}{8}$ is larger than $\frac{1}{4}$).

Gr. 4 NS 1.5: Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; **explain** equivalents of fractions (see Standard 4.0).

Equivalent Fractions

Words to Remember

Equivalent fractions: Two fractions that represent the same value



Getting Started In Lesson 2-4 you learned how to read, write, and represent fractions. You can also write, complete, and identify equivalent fractions.

EXAMPLE 1
Writing Equivalent Fractions

Write three fractions that are equivalent to $\frac{2}{5}$.

Solution

To write an equivalent fraction, multiply the numerator and denominator of a fraction by the same nonzero whole number.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$$

Multiply the numerator and denominator by 2.

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Multiply the numerator and denominator by 3.

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

Multiply the numerator and denominator by 4.

ANSWER The fractions $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{8}{20}$ are equivalent to $\frac{2}{5}$.

TRY THIS

Write three fractions that are equivalent to the given fraction.

1. $\frac{1}{12}$

2. $\frac{3}{7}$

EXAMPLE 2**Completing Equivalent Fractions**

What number will complete the equivalent fraction in $\frac{4}{9} = \frac{12}{?}$?

Solution

Step 1 Think: 4 times what number is 12?

$$\frac{4 \times ?}{9 \times ?} = \frac{12}{?} \quad \text{Write the problem.}$$

$$4 \times 3 = 12 \quad \text{Find the unknown factor.}$$

Step 2 Multiply the denominator 9 by the number you found in Step 1.

$$\frac{4 \times 3}{9 \times 3} = \frac{12}{27}$$

ANSWER 27

TRY THIS

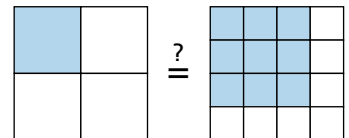
Complete the equivalent fraction.

3. $\frac{5}{6} = \frac{10}{\square}$

4. $\frac{1}{8} = \frac{\square}{56}$

EXAMPLE 3**Identifying Equivalent Fractions**

Tell whether the models represent equivalent fractions.



Solution

$$\frac{1}{4} \stackrel{?}{=} \frac{9}{16}$$

$$\frac{1 \times ?}{4 \times ?} = \frac{9}{16} \rightarrow \frac{1 \times 9}{4 \times 4} = \frac{9}{16} \rightarrow 9 \neq 4$$

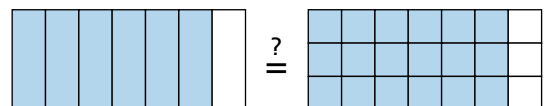
Write the fractions represented by the models.

Determine whether 1 and 4 are multiplied by the same number to get 9 and 16.

ANSWER Since 1 and 4 are multiplied by different numbers to get 9 and 16, the models do not represent equivalent fractions.

TRY THIS

5. Tell whether the models represent equivalent fractions.

**Using Models**

Notice that even though the models are the same size, the total shaded area in the second model appears to be larger than the total shaded area in the first model.

Summarize

Writing Equivalent Fractions

Multiply the numerator and denominator by the same nonzero whole number to write an equivalent fraction.

$$\frac{3}{11} = \frac{3 \times 2}{11 \times 2} = \frac{6}{22}$$

Completing Equivalent Fractions

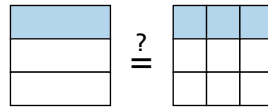
Find the number that when multiplied by the smaller numerator or denominator gives the larger numerator or denominator. The numerator and denominator are multiplied by the same nonzero whole number.

$$\frac{4}{5} = \frac{16}{?}$$

$$\frac{4 \times ?}{5 \times ?} = \frac{16}{?} \rightarrow \frac{4 \times 4}{5 \times ?} = \frac{16}{?} \rightarrow \frac{4 \times 4}{5 \times 4} = \frac{16}{20}$$

Identifying Equivalent Fractions

Determine whether the smaller numerator and denominator are multiplied by the same number to get the larger numerator and denominator.



$$\frac{1}{3} \stackrel{?}{=} \frac{3}{9}$$

$$\frac{1 \times ?}{3 \times ?} = \frac{3}{9} \rightarrow \frac{1 \times 3}{3 \times 3} = \frac{3}{9} \rightarrow 3 = 3, \text{ so } \frac{1}{3} = \frac{3}{9}$$

Practice

Write three fractions that are equivalent to the fraction.

1. $\frac{3}{4}$

2. $\frac{7}{10}$

3. $\frac{1}{6}$

4. $\frac{2}{9}$

5. $\frac{4}{7}$

6. $\frac{11}{12}$

Complete the equivalent fraction.

7. $\frac{1}{10} = \frac{7}{\square}$

8. $\frac{2}{7} = \frac{18}{\square}$

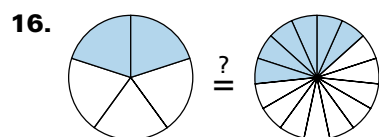
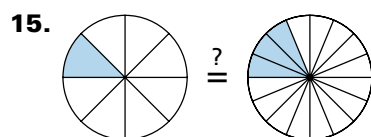
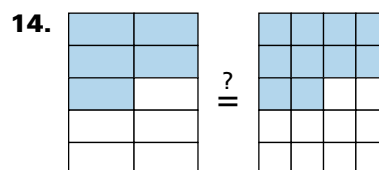
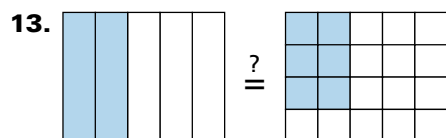
9. $\frac{3}{5} = \frac{6}{\square}$

10. $\frac{8}{9} = \frac{\square}{45}$

11. $\frac{3}{8} = \frac{\square}{16}$

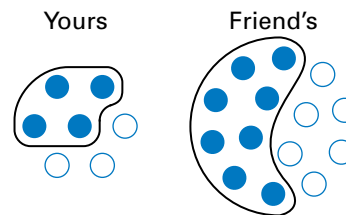
12. $\frac{7}{12} = \frac{\square}{36}$

Tell whether the models represent equivalent fractions.



First tell whether you need to *write, complete, or identify* equivalent fractions. Then solve the problem. Explain your reasoning.

- 17.** The diagram shows your marble collection and a friend's marble collection. Your friend says she has the same fraction of blue marbles in her collection as you have in yours. Is she correct?



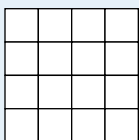
- 18.** Out of 12 students in the math club, 5 are baseball players. The drama club has 36 members. That club has the same fraction of baseball players as the math club. How many baseball players are in the drama club?

- 19.** A deli sells boxes with different kinds of sandwiches. In each box, $\frac{3}{8}$ of the sandwiches are ham and cheese. Suppose there were two boxes of sandwiches, or three boxes, or more. Name three other ways the fraction of ham and cheese sandwiches could be expressed.

DID YOU GET IT?

- 20. Fill in the missing words.** _____ are two fractions that represent the same value.

- 21. Use a model.** About $\frac{3}{4}$ of the blossoms on a tree are fully open. Represent an equivalent fraction by filling in the blank model.



$$\frac{3}{4} = \frac{\boxed{}}{\boxed{}}$$

LESSON
2-7

Simplify Fractions

Words to Remember

Simplest form: The form of a fraction where the greatest common factor of the numerator and denominator is 1

$\frac{5}{8}$ GCF of 5 and 8 is 1.

So, $\frac{5}{8}$ is in simplest form.

$\frac{9}{12}$ GCF of 9 and 12 is 3.

So, $\frac{9}{12}$ is *not* in simplest form.

Getting Started In Lesson 2-2 you learned how to find the greatest common factor (GCF) of two numbers. In Lesson 2-5 you learned how to write equivalent fractions. You can use the GCF of the numerator and the denominator to write an equivalent fraction that is in simplest form.

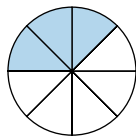
Identifying Fractions in Simplest Form To decide whether a fraction is in simplest form, find the greatest common factor of the numerator and the denominator. If the GCF is 1, the fraction is in simplest form. If it is not 1, then the fraction is not in simplest form.

EXAMPLE 1

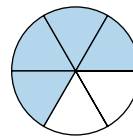
Identifying Fractions in Simplest Form

Tell whether the fraction is in simplest form.

a. $\frac{3}{8}$



b. $\frac{4}{6}$



Solution

a. $\frac{3}{8}$ is in simplest form because the GCF of 3 and 8 is 1.

ANSWER The fraction $\frac{3}{8}$ is in simplest form.

b. $\frac{4}{6}$ is *not* in simplest form because the GCF of 4 and 6 is 2.

ANSWER The fraction $\frac{4}{6}$ is not in simplest form.

TRY THIS

Tell whether the fraction is in simplest form. Write **yes** or **no**.

1. $\frac{4}{16}$ _____

2. $\frac{2}{11}$ _____

Reading

Simplest form is also called *the reduced form* for a fraction.

Simplifying A fraction that is not in simplest form can be written as an equivalent fraction that is in simplest form. To *simplify* a fraction, divide the numerator and the denominator by the GCF.

EXAMPLE 2

Writing Fractions in Simplest Form

Simplify the fraction.

a. $\frac{16}{20}$

b. $\frac{8}{8}$

Solution

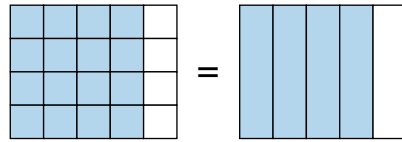
- a. List the factors of 16 and 20.

16: 1, 2, 4, 8, 16

20: 1, 2, 4, 5, 10, 20

The GCF of 16 and 20 is 4.

$$\frac{16 \div 4}{20 \div 4} = \frac{4}{5}$$

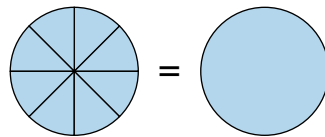


ANSWER $\frac{4}{5}$

- b. List the factors of 8: 1, 2, 4, 8

The GCF of 8 and 8 is 8.

$$\frac{8 \div 8}{8 \div 8} = \frac{1}{1} = 1$$



ANSWER 1

Find the GCF of 16 and 20.

Divide 16 and 20 by 4.

Look at models that represent $\frac{16}{20}$ and $\frac{4}{5}$. The shaded parts in each model are the same size. That is how you know the fractions are equivalent.

Find the GCF of 8 and 8.

Divide 8 and 8 by the GCF.


Look at the model that represents $\frac{8}{8}$. All the parts are shaded. So, 8 parts of 8 is equal to one whole, or 1.


Remember

When you simplify $\frac{16}{20}$, you are writing an equivalent fraction. The equivalent fraction has the smallest possible numbers in the numerator and denominator.

TRY THIS

Simplify the fraction.

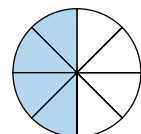
3. $\frac{14}{26} \rightarrow$ 

4. $\frac{11}{11} \rightarrow$ 

5. Which fraction represents the simplest form of the model? _____

A. $\frac{1}{4}$

B. $\frac{1}{2}$



Summarize**Identifying Fractions in Simplest Form**

Find the GCF of the numerator and denominator.

Simplifying Fractions

Divide the numerator and denominator by the GCF.

$$\frac{5}{9} \quad \text{GCF is 1. } \rightarrow 1 = 1$$

So, $\frac{5}{9}$ is in simplest form.

$$\frac{6}{8} \quad \text{GCF is 2. } \rightarrow 2 \neq 1$$

So, $\frac{6}{8}$ is not in simplest form.

$$\frac{7}{21} \quad \frac{7 \div 7}{21 \div 7} = \frac{1}{3}$$

$$\frac{3}{3} \quad \frac{3 \div 3}{3 \div 3} = \frac{1}{1} = 1$$

PracticeTell whether the fraction or model is in simplest form. Write *yes* or *no*.

1. $\frac{7}{12}$ _____

2. $\frac{6}{9}$ _____

3. $\frac{5}{5}$ _____

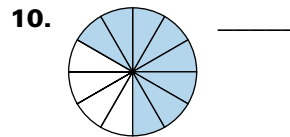
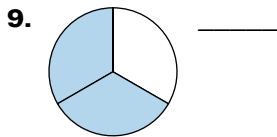
4. $\frac{8}{15}$ _____

5. $\frac{2}{5}$ _____


6. $\frac{3}{7}$ _____


7. $\frac{18}{24}$ _____


8. $\frac{25}{50}$ _____





Simplify the fraction.


11. $\frac{3}{9} \rightarrow$ 


12. $\frac{8}{18} \rightarrow$ 


13. $\frac{6}{15} \rightarrow$ 


14. $\frac{12}{28} \rightarrow$ 


15. $\frac{30}{40} \rightarrow$ 


16. $\frac{24}{36} \rightarrow$ 


17. $\frac{14}{49} \rightarrow$ 

18. $\frac{10}{100} \rightarrow$ 

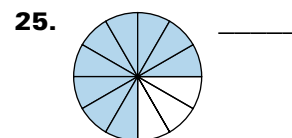
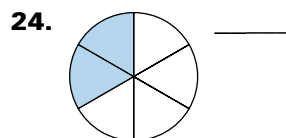
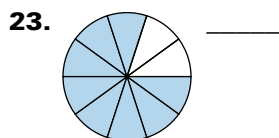
19. $\frac{27}{60} \rightarrow$ 

20. $\frac{25}{90} \rightarrow$ 

21. $\frac{55}{65} \rightarrow$ 

22. $\frac{48}{96} \rightarrow$ 

Match each model with the equivalent fraction in simplest form.



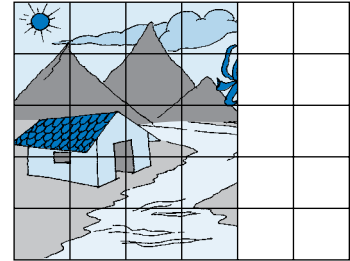
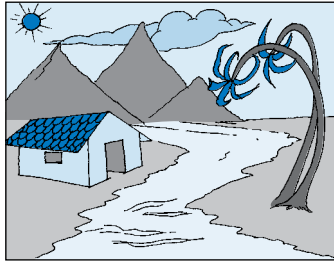
A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{1}{3}$

Write a fraction to represent the situation. Then simplify the fraction to answer the question.

- 26.** You are copying a picture for art class. The diagram shows how much you have finished so far. What fraction of the picture have you drawn?



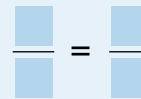
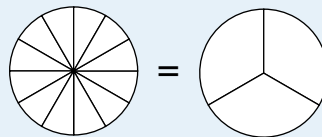
- 27.** You have completed **18** of the **24** problems on a quiz. What fraction of the quiz have you completed?

- 28.** After school, you studied for a total of **25** minutes. You spent **10** minutes working on math homework. What fraction of your study time was spent on math homework?

DID YOU GET IT?

- 29. Fill in the missing words.** When a fraction is in _____, the GCF of the numerator and denominator is **1**.

- 30. Use a model.** Use the blank models at the right to represent a fraction that is not in simplest form and an equivalent fraction that is in simplest form.



- 31. Explain your reasoning.** You have read **15** pages of a **20**-page short story. You tell your friend that you are $\frac{3}{4}$ of the way through the story. Is your statement correct? Explain.

LESSON
2-7

California
Standards

Gr. 2 NS 4.1: Recognize, name, and **compare** unit fractions from $\frac{1}{12}$ to $\frac{1}{2}$.

Gr. 6 NS 1.1: Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

Compare and Order Fractions

Words to Remember

Least common denominator (LCD): The least common multiple (LCM) of the denominators of two or more fractions

For example: $\frac{3}{4}$ and $\frac{1}{6}$ ← **denominators**

4, 8, 12, 16, 20, ...

6, 12, 18, 24, ...

least common multiple (LCM) = 12

The LCD of $\frac{3}{4}$ and $\frac{1}{6}$ is 12. **LCM = LCD**

Getting Started In Lesson 2-3 you learned how to find the least common multiple. You can use the least common multiple of two or more denominators to compare and order fractions. The least common multiple of the denominators of two or more fractions is also known as the *least common denominator*, or LCD.

EXAMPLE 1 Finding the LCD

Find the least common denominator of $\frac{3}{8}$ and $\frac{5}{12}$.

Solution

Step 1 Identify the denominators of the fractions.

$$\frac{3}{8}$$

$$\frac{5}{12}$$

The denominators are **8** and **12**.

Step 2 Find the least common multiple of the denominators.

Start by listing the multiples for each number.

8, 16, **24**, 32, 40, ...

12, **24**, 36, 48, 60, ...

The LCM of **8** and **12** is **24**.

Step 3 Find the least common multiple of the denominators. This number is the least common denominator (LCD).

ANSWER The LCD of $\frac{3}{8}$ and $\frac{5}{12}$ is **24**.

TRY THIS Find the LCD.

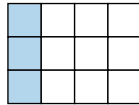
1. $\frac{2}{9}$ and $\frac{3}{9}$ _____

2. $\frac{4}{5}$ and $\frac{7}{10}$ _____

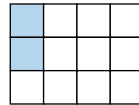
EXAMPLE 2**Comparing Fractions Using the LCD**Compare $\frac{1}{4}$ and $\frac{1}{6}$.**Solution****Step 1** Find the least common denominator of $\frac{1}{4}$ and $\frac{1}{6}$.4, 8, **12**, 16, 20, ... 6, **12**, 18, 24, 30, ... The LCD is 12.**Step 2** Use the LCD to write equivalent fractions:**Using a Model**


Notice that there is more shading in the model of $\frac{3}{12}$ than in the model of $\frac{2}{12}$.


$$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$



$$\frac{1 \times 2}{6 \times 2} = \frac{2}{12}$$

**Step 3** Notice that $3 > 2$, so $\frac{3}{12} > \frac{2}{12}$.**ANSWER** You know that $\frac{3}{12} > \frac{2}{12}$, so $\frac{1}{4} > \frac{1}{6}$.**TRY THIS**Compare the fractions. Complete the statement with $<$, $>$, or $=$.

3. $\frac{5}{8}$  $\frac{3}{4}$

4. $\frac{5}{6}$  $\frac{6}{7}$

EXAMPLE 3**Ordering Fractions Using the LCD**Order the fractions $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{15}$ from least to greatest.**Solution****Step 1** Use the LCD to write equivalent fractions. The LCD of $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{15}$ is 15.

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$

$$\frac{7 \times 1}{15 \times 1} = \frac{7}{15}$$

Step 2 Notice that $7 < 9 < 10$, so $\frac{7}{15} < \frac{9}{15} < \frac{10}{15}$ or $\frac{7}{15} < \frac{3}{5} < \frac{2}{3}$.**ANSWER** The fractions in order from least to greatest are $\frac{7}{15}$, $\frac{3}{5}$, and $\frac{2}{3}$.**TRY THIS**

Order the fractions from least to greatest.

5. $\frac{1}{9}$, $\frac{1}{4}$, and $\frac{1}{12}$

6. $\frac{1}{6}$, $\frac{7}{8}$, and $\frac{2}{3}$

Summarize

Finding the Least Common Denominator

Find the LCM of the denominators. The LCM of the denominators is the least common denominator.

Comparing Fractions

Use the LCD to write equivalent fractions. Then compare the fractions.

Ordering Fractions

Use the LCD to write equivalent fractions. Then order the fractions.

Practice

Find the least common denominator of the fractions.

1. $\frac{1}{2}$ and $\frac{11}{14}$ _____

2. $\frac{5}{6}$ and $\frac{4}{9}$ _____

3. $\frac{3}{5}$ and $\frac{1}{7}$ _____

4. $\frac{2}{3}$ and $\frac{5}{8}$ _____

5. $\frac{7}{15}$ and $\frac{3}{4}$ _____

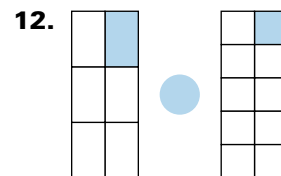
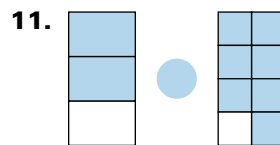
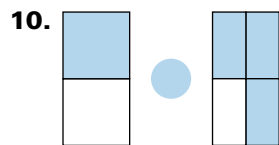
6. $\frac{9}{10}$ and $\frac{5}{6}$ _____


7. $\frac{1}{4}$, $\frac{5}{14}$, and $\frac{3}{28}$ _____


8. $\frac{1}{2}$, $\frac{3}{10}$, and $\frac{9}{16}$ _____


9. $\frac{1}{6}$, $\frac{7}{15}$, and $\frac{4}{5}$ _____


Complete the statement with $<$, $>$, or $=$.





13. $\frac{8}{11}$  $\frac{10}{11}$

14. $\frac{1}{2}$  $\frac{3}{6}$

15. $\frac{2}{3}$  $\frac{4}{9}$

16. $\frac{3}{8}$  $\frac{2}{7}$

17. $\frac{7}{10}$  $\frac{4}{5}$

18. $\frac{9}{20}$  $\frac{5}{12}$

Order the fractions from least to greatest.

19. $\frac{1}{9}$, $\frac{1}{3}$, and $\frac{1}{12}$

20. $\frac{4}{7}$, $\frac{3}{4}$, and $\frac{5}{8}$

21. $\frac{5}{6}$, $\frac{9}{10}$, and $\frac{5}{12}$

22. $\frac{7}{9}$, $\frac{5}{6}$, and $\frac{5}{18}$

23. $\frac{7}{10}$, $\frac{4}{5}$, and $\frac{3}{20}$

24. $\frac{11}{16}$, $\frac{3}{8}$, and $\frac{13}{32}$

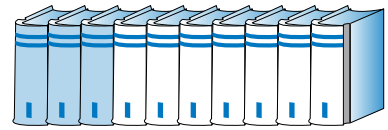
25. $\frac{1}{4}$, $\frac{5}{16}$, and $\frac{4}{15}$

26. $\frac{9}{20}$, $\frac{5}{12}$, and $\frac{3}{5}$

27. $\frac{6}{7}$, $\frac{7}{9}$, and $\frac{3}{4}$

Solve. Explain your answer.

- 28.** The diagram shows the number of books you have read on your summer reading list. Your friend has read $\frac{2}{5}$ of the books on the list. Who has read more books on the list?



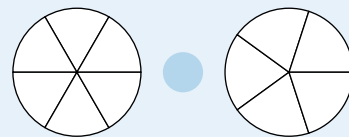
- 29.** You are sewing lace onto some pillows. The lengths of lace that you need for three pillows are $\frac{7}{9}$ yards, $\frac{3}{4}$ yards, and $\frac{5}{6}$ yards. Order these measurements from least to greatest.

- 30.** You are building a shelf over a fireplace. You have just attached a $\frac{7}{12}$ -inch piece of wood to the shelf. The next piece of wood you attach needs to be slightly shorter. Should you use a piece that measures $\frac{9}{16}$ inches or $\frac{5}{8}$ inches?

DID YOU GET IT?

- 31. Fill in the missing words.** The _____ of two fractions is the least common multiple of the denominators of the fractions.

- 32. Use a model.** Use the blank models at the right to write a comparison statement with $<$ or $>$. Then write the comparison using numbers.



LESSON 2-8



California Standards

Gr. 4 NS 1.5:

Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; **explain equivalents of fractions** (see Standard 4.0).

Gr. 4 NS 1.6: Write tenths and hundredths in decimal and **fraction notations** and know the **fraction** and decimal **equivalents for halves and fourths** (e.g., $\frac{1}{2} = 0.5$ or 0.50 ; $\frac{7}{4} = 1\frac{3}{4} = 1.75$).

Mixed Numbers and Improper Fractions

Words to Remember

Mixed number: A number that represents the sum of a whole number and a fraction

Improper fraction: A fraction with a numerator that is greater than or equal to the denominator

$$\text{Mixed number: } 5 + \frac{1}{4} = 5\frac{1}{4}$$

Whole number part **Fraction part**

$$\text{Improper fraction: } \frac{10}{3}$$

$$10 > 3$$

numerator > denominator

Getting Started In Lesson 2-4 you learned how to read, write, and represent fractions. You can also read, write, and represent mixed numbers and improper fractions.

EXAMPLE 1

Writing Improper Fractions as Mixed Numbers

Write $\frac{13}{5}$ as a mixed number.

Solution

Step 1 Divide 13 by 5.

$$\begin{array}{r} 2 \leftarrow \text{Whole number part} \\ \text{Denominator} \rightarrow 5 \overline{)13} \\ \underline{10} \\ 3 \leftarrow \text{Numerator in fraction part} \end{array}$$

$$13 \div 5 = 2 \text{ R } 3$$

Step 2 Write the mixed number. The quotient, or answer, is the whole number part of the mixed number. The remainder becomes the numerator of the fraction part of the mixed number. Keep the denominator from the improper fraction.

$$\text{ANSWER } 2\frac{3}{5}$$

TRY THIS

Write the improper fraction as a mixed number.

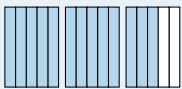
$$1. \frac{7}{2} = \square \frac{\square}{\square}$$

$$2. \frac{16}{9} = \square \frac{\square}{\square}$$

Using a Model

You can use a model to represent an improper fraction.

Shade the model the same way you would shade a model for a proper fraction. Each whole has 5 equal parts. Shade 13 of the parts.



There are 2 wholes plus $\frac{3}{5}$. So, $\frac{13}{5} = 2\frac{3}{5}$.

EXAMPLE 2**Writing Mixed Numbers as Improper Fractions**

Write $6\frac{3}{4}$ as an improper fraction.

Solution

Step 1 Multiply the whole number by the denominator. Then add the numerator. The sum becomes the numerator of the improper fraction.

$$6\frac{3}{4} \quad (6 \times 4) + 3 = 27$$

Step 2 Write the improper fraction. Keep the denominator from the mixed number.

ANSWER $\frac{27}{4}$

TRY THIS

Write the mixed number as an improper fraction.

$$3. \quad 3\frac{1}{6} = \frac{(\square \times \square) + \square}{\square} = \frac{\square + \square}{\square} = \frac{\square}{\square}$$

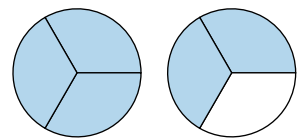
$$4. \quad 1\frac{4}{7} = \frac{(\square \times \square) + \square}{\square} = \frac{\square + \square}{\square} = \frac{\square}{\square}$$

EXAMPLE 3**Using a Model**

Use a model to write $1\frac{2}{3}$ as an improper fraction.

Solution

Step 1 Model $1\frac{2}{3}$ by shading 1 circle and $\frac{2}{3}$ of another circle.

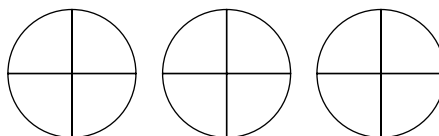


Step 2 Count the total number of shaded parts. This is the numerator of the improper fraction. The number of equal parts in each circle is the denominator. There are 5 shaded parts and 3 equal parts in each circle.

ANSWER $\frac{5}{3}$

TRY THIS

5. Use the model to write $2\frac{1}{4}$ as an improper fraction.



Summarize

Writing Improper Fractions as Mixed Numbers

Divide the numerator by the denominator. The quotient is the whole number part of the mixed number. The remainder is the numerator of the fraction part of the mixed number. The denominator stays the same.

$$\frac{11}{2} \rightarrow 2 \overline{)11} \rightarrow 5 \frac{1}{2}$$

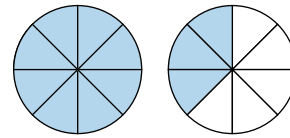
Writing Mixed Numbers as Improper Fractions

Multiply the whole number by the denominator. Then add the numerator. This becomes the numerator of the improper fraction. Keep the denominator the same.

$$3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$$

Using Models

The model represents a mixed number, $1\frac{3}{8}$, and an improper fraction, $\frac{11}{8}$.



Practice

Write the improper fraction as a mixed number.

1. $\frac{12}{7}$

2. $\frac{14}{5}$

3. $\frac{15}{2}$

4. $\frac{8}{3}$

5. $\frac{7}{4}$

6. $\frac{17}{6}$

7. $\frac{21}{10}$

8. $\frac{25}{9}$

Write the mixed number as an improper fraction.

9. $2\frac{7}{8}$

10. $5\frac{1}{4}$

11. $3\frac{2}{3}$

12. $1\frac{4}{9}$

13. $4\frac{3}{7}$

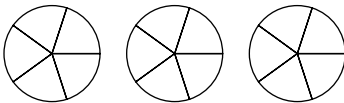
14. $3\frac{2}{5}$

15. $5\frac{1}{10}$

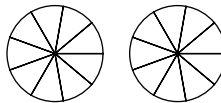
16. $6\frac{5}{12}$

Use a model to write the mixed number as an improper fraction.

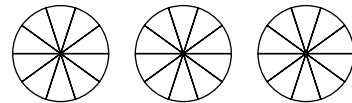
17. $2\frac{2}{5} = \frac{\boxed{}}{\boxed{}}$



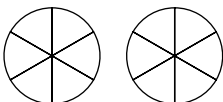
18. $1\frac{1}{9} = \frac{\boxed{}}{\boxed{}}$



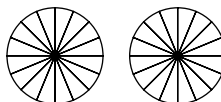
19. $2\frac{3}{10} = \frac{\boxed{}}{\boxed{}}$



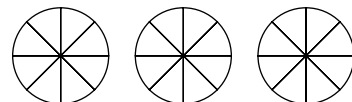
20. $1\frac{5}{6} = \frac{\boxed{}}{\boxed{}}$



21. $1\frac{7}{16} = \frac{\boxed{}}{\boxed{}}$



22. $2\frac{5}{8} = \frac{\boxed{}}{\boxed{}}$

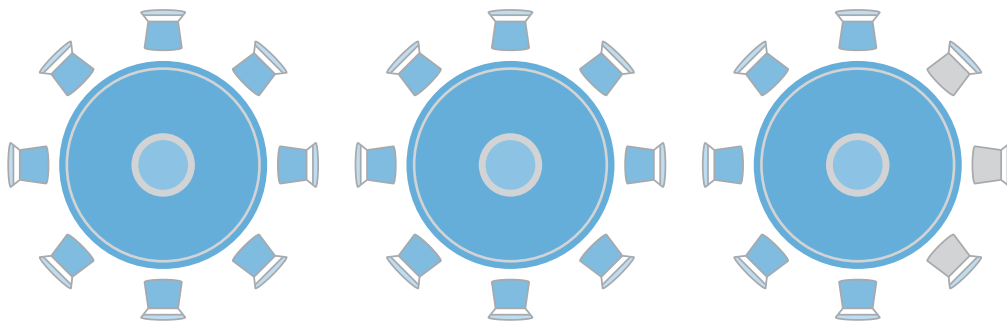


Solve. Show all your work.

- 23.** Thom served $2\frac{1}{2}$ pitchers of punch at a birthday party. Write this amount as an improper fraction.
- _____

- 24.** A recipe for pizza dough needs $\frac{11}{2}$ cups of flour. What is $\frac{11}{2}$ written as a mixed number?
- _____

- 25.** The diagram shows the number of seats that were filled at a dinner party. Write the amount as a mixed number.

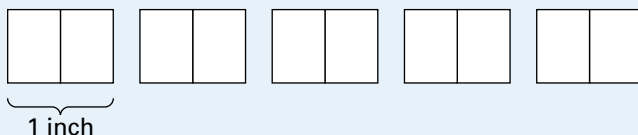
**DID YOU GET IT?**

- 26. Fill in the missing words.** A(n) _____ represents the sum of a whole number and a fraction. In a(n) _____, the numerator is greater than or equal to the denominator.

- 27. Find the error.** Stacy shows her steps as she writes $5\frac{4}{9}$ as an improper fraction. Describe and correct the error in Stacy's work.

$$5\frac{4}{9} = \frac{5 \times 4}{9} = \frac{20}{9}$$

- 28. Use a model.** There are $4\frac{1}{2}$ inches of snow on the ground. Fill in the blank model to represent this situation. Then write the amount as an improper fraction.



LESSON
2-9

California Standards

Gr. 6 NS 1.1: Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

Gr. 3/4/5/6/7 MR 2.2: Apply strategies and results from simpler problems to more complex problems.

Compare and Order Mixed Numbers and Improper Fractions

Getting Started In Lesson 2-7 you learned how to compare and order fractions. You can also compare and order mixed numbers and improper fractions.

EXAMPLE 1
Comparing Mixed Numbers

Compare the mixed numbers.

a. $2\frac{8}{9}$ and $1\frac{11}{12}$

b. $4\frac{3}{5}$ and $4\frac{7}{15}$

Solution

a. Compare the whole numbers.

ANSWER You know that $2 > 1$, so $2\frac{8}{9} > 1\frac{11}{12}$.

b. Compare the fraction parts. Use the LCD to write equivalent fractions.

$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \qquad \frac{7}{15}$$

ANSWER You know that $\frac{9}{15} > \frac{7}{15}$, so $\frac{3}{5} > \frac{7}{15}$. That means that $4\frac{3}{5} > 4\frac{7}{15}$.

Take Note!

In part (b), the whole number parts are equal, so compare the fraction parts.

EXAMPLE 2
Comparing Improper Fractions

Compare $\frac{9}{4}$ and $\frac{12}{5}$.

Solution

Use the LCD to write equivalent fractions.

$$\frac{9}{4} = \frac{9 \times 5}{4 \times 5} = \frac{45}{20}$$

$$\frac{12}{5} = \frac{12 \times 4}{5 \times 4} = \frac{48}{20}$$

ANSWER You know that $\frac{45}{20} < \frac{48}{20}$, so $\frac{9}{4} < \frac{12}{5}$.

TRY THIS

Complete the statement with $<$ or $>$.

1. $1\frac{4}{9}$ \bullet $1\frac{13}{18}$

2. $\frac{7}{2}$ \bullet $\frac{10}{3}$

EXAMPLE 3**Comparing Mixed Numbers and Improper Fractions**Compare $1\frac{3}{4}$ and $\frac{15}{8}$.**Solution****Step 1** Write the mixed number as an improper fraction.


$$1\frac{3}{4} = \frac{4 \times 1 + 3}{4} = \frac{7}{4}$$


Step 2 Use the LCD to write equivalent fractions.

$$\frac{7}{4} = \frac{7 \times 2}{4 \times 2} = \frac{14}{8} \qquad \frac{15}{8}$$

ANSWER You know that $\frac{14}{8} < \frac{15}{8}$, so $1\frac{3}{4} < \frac{15}{8}$.**TRY THIS**

Compare the numbers.

3. $3\frac{5}{8}$  $\frac{11}{3}$

4. $\frac{21}{5}$  $4\frac{2}{11}$

EXAMPLE 4**Ordering Mixed Numbers and Improper Fractions**Order $\frac{17}{3}$, $3\frac{2}{3}$, and $3\frac{5}{9}$ from least to greatest.**Solution****Step 1** Write the mixed numbers as improper fractions.

$$3\frac{2}{3} = \frac{3 \times 3 + 2}{3} = \frac{11}{3} \qquad 3\frac{5}{9} = \frac{9 \times 3 + 5}{9} = \frac{32}{9}$$

Step 2 Use the LCD to write equivalent fractions.

$$\frac{17}{3} = \frac{17 \times 3}{3 \times 3} = \frac{51}{9} \qquad \frac{11}{3} = \frac{11 \times 3}{3 \times 3} = \frac{33}{9} \qquad \frac{32}{9}$$

You know that $\frac{32}{9} < \frac{33}{9}$ and $\frac{33}{9} < \frac{51}{9}$, so $3\frac{5}{9} < 3\frac{2}{3}$ and $3\frac{2}{3} < \frac{17}{3}$.**ANSWER** The numbers in order from least to greatest are $3\frac{5}{9}$, $3\frac{2}{3}$, and $\frac{17}{3}$.**TRY THIS**

Order the numbers from least to greatest.

5. $2\frac{7}{12}$, $2\frac{1}{6}$, and $\frac{41}{24}$

6. $3\frac{4}{5}$, $\frac{67}{20}$, and $\frac{33}{10}$

Summarize**Comparing Mixed Numbers**

Compare whole numbers. If they are equal, compare fraction parts.

Comparing Improper Fractions


Use the LCD to write equivalent fractions. Then compare the fractions.


Comparing and Ordering Mixed Numbers and Improper Fractions


Write the mixed numbers as improper fractions. Use the LCD to write equivalent fractions. Then compare and order the fractions.


Practice


Complete the statement with $<$, $>$, or $=$.


1. $3\frac{1}{3}$  $2\frac{5}{6}$


2. $7\frac{9}{10}$  $7\frac{4}{5}$


3. $6\frac{3}{8}$  $6\frac{2}{3}$


4. $\frac{19}{8}$  $\frac{11}{4}$

5. $\frac{23}{6}$  $\frac{29}{4}$

6. $\frac{16}{9}$  $\frac{17}{11}$

7. $1\frac{5}{6}$  $\frac{23}{12}$

8. $\frac{11}{5}$  $2\frac{1}{10}$

9. $4\frac{2}{3}$  $\frac{51}{12}$

Circle the greatest value.

10. $\frac{12}{7}$ $\frac{25}{14}$ $1\frac{9}{14}$

11. $\frac{9}{2}$ $4\frac{1}{3}$ $\frac{17}{4}$

12. $8\frac{4}{5}$ $\frac{91}{10}$ $9\frac{2}{5}$

13. $\frac{14}{9}$ $1\frac{5}{8}$ $1\frac{11}{20}$

14. $2\frac{1}{8}$ $\frac{31}{16}$ $\frac{41}{20}$

15. $5\frac{3}{10}$ $\frac{59}{12}$ $\frac{35}{6}$

Order the numbers from least to greatest.

16. $3\frac{1}{16}$, $\frac{9}{4}$, $\frac{41}{16}$

17. $\frac{13}{8}$, $1\frac{7}{12}$, $\frac{17}{12}$

18. $\frac{26}{9}$, $\frac{8}{3}$, $2\frac{5}{6}$

19. $\frac{8}{3}$, $2\frac{1}{2}$, $\frac{18}{7}$

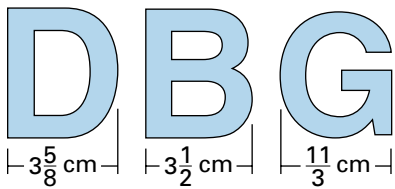
20. $\frac{41}{10}$, $4\frac{2}{15}$, $\frac{49}{12}$

21. $5\frac{7}{12}$, $\frac{43}{8}$, $\frac{16}{3}$

Solve. Explain your answer.

- 22.** You use $3\frac{3}{4}$ cups of flour to bake bread and $3\frac{7}{16}$ cups of flour to bake a cake. Do you use more flour for the bread or the cake?

- 23.** Order the letters from narrowest to widest.



- 24.** Sam walked $\frac{8}{3}$ miles. Jamila walked $\frac{11}{9}$ miles. Who walked farther?

DID YOU GET IT?

- 25. Fill in the missing words.** To compare a mixed number and an improper fraction, first write the _____ as a(n) _____. Then use the _____ to write equivalent fractions.
- 26. Explain your reasoning.** You are working on a painting for art class. To fit in a frame, the length of the canvas needs to be between $8\frac{5}{12}$ inches and $\frac{35}{4}$ inches. You cut your canvas so that it measures $8\frac{2}{3}$ inches. Will the canvas fit in the frame? Explain.

Mixed Practice for Lessons 2-4 to 2-9

Vocabulary Review

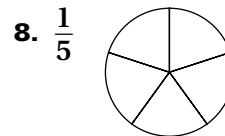
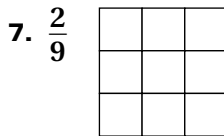
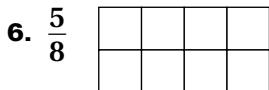
Match the word with its mathematical meaning and its everyday meaning.

Word	Mathematical meaning	Everyday meaning
1. simplify __, __	A. describes a fraction with a numerator greater than the denominator	X. smaller part of a larger group
2. improper __, __	B. to write a fraction in simplest form	Y. not correct
3. fraction __, __	C. a number written in the form $\frac{a}{b}$ that refers to parts of a set or parts of a whole	Z. to make easier

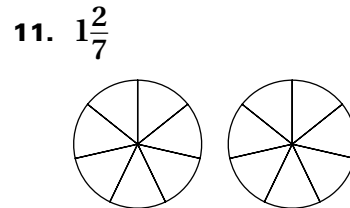
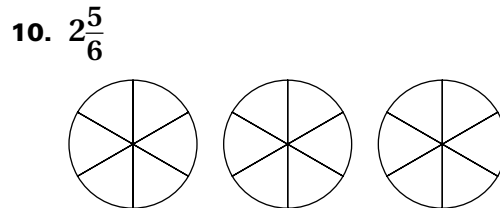
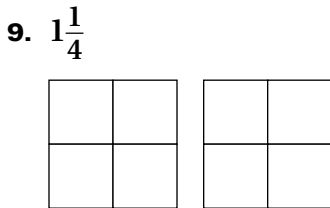
Fill in the missing word(s).

4. The fractions $\frac{3}{7}$ and $\frac{6}{21}$ are _____ because they represent the same value.
5. The _____ of a fraction refers to the whole, or number of equal parts in a set.

Identify the numerator and denominator of the fraction. Then shade in the model to represent the fraction.



Use the model to write the mixed number as an improper fraction.



Complete the equivalent fraction.

12. $\frac{2}{3} = \frac{6}{\quad}$
13. $\frac{1}{10} = \frac{\quad}{40}$
14. $\frac{3}{5} = \frac{\quad}{25}$
15. $\frac{6}{7} = \frac{18}{\quad}$

Order the numbers from least to greatest.

16. $\frac{3}{7}, \frac{3}{8}, \frac{4}{9}$
17. $3\frac{5}{6}, \frac{25}{6}, 3\frac{2}{3}$
18. $\frac{11}{4}, 2\frac{7}{8}, \frac{21}{8}$

19. Fill in the missing information to solve the problem.

Last year, sixth graders at a school spent an average of $\frac{3}{20}$ of each school day getting exercise. The principal wants to reorganize the school day next year to make more time for exercise. What fraction of the day should the school allow for exercise, $\frac{2}{9}$ or $\frac{1}{8}$?

Step 1 The phrase “more time” tells you that you need to compare the fractions $\frac{2}{9}$ and $\frac{1}{8}$ to the fraction $\frac{3}{20}$ to tell which one is _____.

Step 2 Compare: $\frac{2}{9} \bigcirc \frac{3}{20}$ and $\frac{1}{8} \bigcirc \frac{3}{20}$.

Step 3 The school should allow _____ of the day for exercise.

Simplify the fractions. Write any improper fractions as mixed numbers.

20. $\frac{6}{15}$

21. $\frac{9}{36}$

22. $\frac{8}{28}$

23. $\frac{16}{40}$

24. $\frac{26}{9}$

25. $\frac{33}{8}$

26. $\frac{14}{3}$

27. $\frac{65}{12}$

Complete the statement with <, >, or =.

28. $\frac{4}{5} \bigcirc \frac{6}{7}$

29. $\frac{2}{9} \bigcirc \frac{4}{11}$

30. $\frac{9}{4} \bigcirc \frac{8}{5}$

31. $\frac{11}{3} \bigcirc 3\frac{1}{9}$

32. $4\frac{3}{4} \bigcirc 4\frac{3}{5}$

33. $1\frac{1}{6} \bigcirc \frac{8}{3}$

34. $3\frac{1}{10} \bigcirc 2\frac{11}{12}$

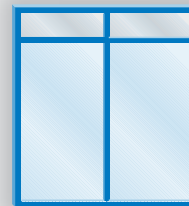
35. $\frac{21}{8} \bigcirc \frac{17}{4}$

Solve the problem. Explain your reasoning.

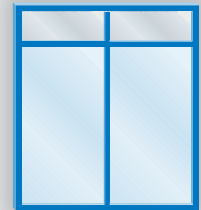
36. Three windows are on display at a store. The heights of the windows are $3\frac{7}{10}$ feet, $3\frac{4}{5}$ feet, and $\frac{41}{20}$ feet. Label each window in the diagram with its height.



Height _____



Height _____



Height _____

37. Jack is about $\frac{3}{8}$ of the way finished with his science project. Alexandra is about $\frac{5}{12}$ of the way finished with her science project. Who is farther along?

ACTIVITY 2-10



California Standards

Gr. 3 NS 3.1: Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $\frac{1}{2}$ of a pizza is the same amount as $\frac{2}{4}$ of another pizza that is the same size; show that $\frac{3}{8}$ is larger than $\frac{1}{4}$).

Gr. K MR 1.2: Use tools and strategies, such as manipulatives and sketches, to model problems.

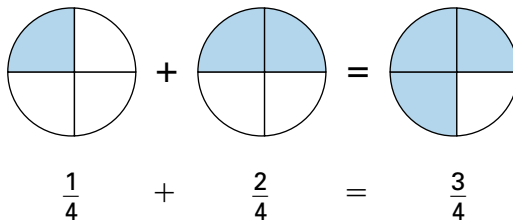
Also included: Gr. 3/4/5 MR 2.3, Gr. 6 MR 2.4, and Gr. 7 MR 2.5

Adding Fractions Using Models

Goal: Use models to understand how to add fractions.

Materials: Addition Expressions list

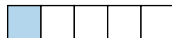
Getting Started In Lesson 2-4 you learned how to represent a fraction using a model. You can also use models to add fractions.



EXAMPLE

Use the following steps to model the sum: $\frac{1}{5} + \frac{3}{5}$

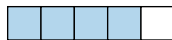
Step 1 Model $\frac{1}{5}$ by shading 1 out of 5 squares.



Step 2 Model $\frac{3}{5}$ by shading 3 out of 5 squares.



Step 3 Combine the models by shading 4 out of 5 squares.

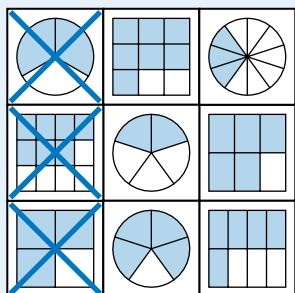


Step 4 Write the result.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

MAKE IT A GAME!

- Form groups of three or four students. Choose a caller.
- Give the caller the list of addition expressions. All other group members get a bingo card.
- The caller will call out the expressions at random. When an expression is called out, players look for a model on their card that represents the sum of the expression. If the model is there, the players mark it with an X.
- The first player in each group that marks three correct models in a row, column, or diagonal wins.



The caller called out:

$$\frac{1}{3} + \frac{1}{3}$$

$$\frac{5}{12} + \frac{2}{12}$$

$$\frac{2}{4} + \frac{1}{4}$$

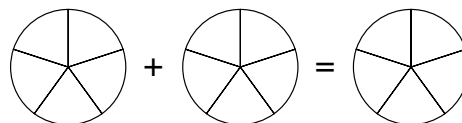
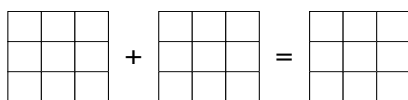
So, this player wins.

Practice

In Exercises 1 and 2, use models to find the sum.

1. $\frac{6}{9} + \frac{1}{9} = \frac{\boxed{}}{\boxed{}}$

2. $\frac{1}{5} + \frac{1}{5} = \frac{\boxed{}}{\boxed{}}$



In Exercises 3–6, draw models to find the sum.

3. $\frac{1}{4} + \frac{2}{4}$

4. $\frac{1}{3} + \frac{1}{3}$

5. $\frac{2}{10} + \frac{5}{10}$

6. $\frac{5}{12} + \frac{6}{12}$

7. **Make a Conjecture** Based on your answers to Exercises 1–6, make a conjecture about how to find the sum of two fractions without using models.

DID YOU GET IT?

Use your conjecture to find the sum.

8. $\frac{2}{5} + \frac{2}{5} = \frac{\boxed{}}{\boxed{}}$

9. $\frac{1}{9} + \frac{4}{9} = \frac{\boxed{}}{\boxed{}}$

LESSON 2-11



California Standards

Gr. 3 NS 3.1: Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., $\frac{1}{2}$ of a pizza is the same amount as $\frac{2}{4}$ of another pizza that is the same size; show that $\frac{3}{8}$ is larger than $\frac{1}{4}$).

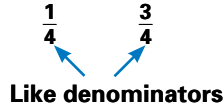
Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Also included: Gr. 3 NS 3.2

Add and Subtract Fractions with Like Denominators

Words to Remember

Like denominators: Denominators that are equal



Getting Started In Activity 2-10 you learned how to add fractions using models. In this lesson you will learn to add and subtract fractions without using models.

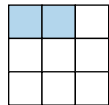
EXAMPLE 1

Adding Fractions Using Models

Use models to find the sum: $\frac{2}{9} + \frac{5}{9}$

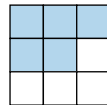
Solution

Step 1 Model $\frac{2}{9}$ by shading 2 out of 9 squares.



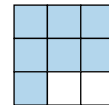
+

Step 2 Model $\frac{5}{9}$ by shading 5 out of 9 squares.



=

Step 3 Model the sum by shading 7 out of 9 squares.



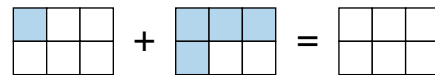
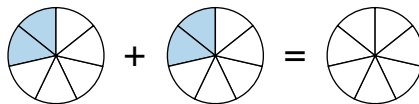
ANSWER $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$

TRY THIS

Use models to find the sum.

1. $\frac{2}{7} + \frac{2}{7} = \frac{\square}{\square}$

2. $\frac{1}{6} + \frac{4}{6} = \frac{\square}{\square}$



Adding Fractions To add fractions with like denominators, write the sum of the numerators over the denominator.

$$\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$$

Simplifying the Sum If the answer is not in simplest form, simplify the fraction. If the answer is an improper fraction, write it as a mixed number.

EXAMPLE 2

Adding Fractions with Like Denominators

Find the sum: $\frac{3}{5} + \frac{4}{5}$

Solution

$$\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5}$$

Write the sum of the numerators over the denominator.

$$= \frac{7}{5}$$

Add.

$$= 1\frac{2}{5}$$

Write the improper fraction as a mixed number.

TRY THIS

Find the sum.

$$3. \frac{3}{8} + \frac{5}{8} = \frac{\square + \square}{\square} = \frac{\square}{\square} = \square \quad 4. \frac{8}{16} + \frac{11}{16} = \frac{\square + \square}{\square} = \frac{\square}{\square} = \square \frac{\square}{\square}$$

Subtracting Fractions To subtract two fractions with like denominators, write the difference of the numerators over the denominator.

$$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$$

EXAMPLE 3

Subtracting Fractions with Like Denominators

Find the difference: $\frac{5}{6} - \frac{1}{6}$

Solution

$$\frac{5}{6} - \frac{1}{6} = \frac{5-1}{6}$$

Write the difference of the numerators over the denominator.

$$= \frac{4}{6}$$

Subtract.

$$= \frac{2}{3}$$

Simplify.

TRY THIS

Find the difference.

$$5. \frac{6}{7} - \frac{2}{7} = \frac{\square - \square}{\square} = \frac{\square}{\square} \quad 6. \frac{7}{8} - \frac{3}{8} = \frac{\square - \square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

Summarize

Adding Fractions with Like Denominators

Write the sum of the numerators over the denominator.
Simplify if possible.

$$\frac{3}{7} + \frac{5}{7} = \frac{3+5}{7} = \frac{8}{7} = 1\frac{1}{7}$$

Subtracting Fractions with Like Denominators

Write the difference of the numerators over the denominator. Simplify if possible.

$$\frac{7}{10} - \frac{3}{10} = \frac{7-3}{10} = \frac{4}{10} = \frac{2}{5}$$

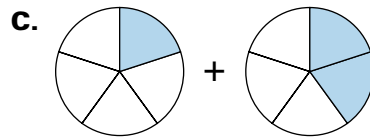
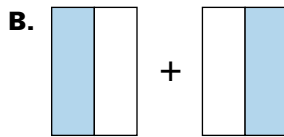
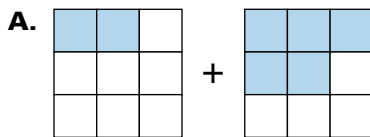
Practice

Match the addition problem with the model that represents it.


1. $\frac{1}{2} + \frac{1}{2}$ _____


2. $\frac{1}{5} + \frac{2}{5}$ _____

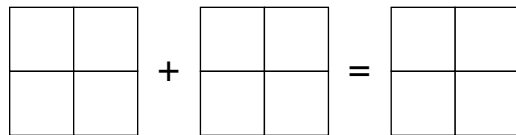
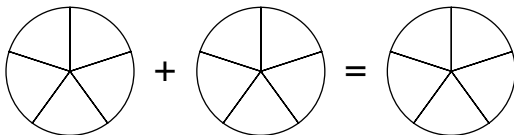
3. $\frac{2}{9} + \frac{5}{9}$ _____



Shade in the models below to find the sum of the fractions.

4. $\frac{3}{5} + \frac{1}{5} =$ 

5. $\frac{3}{4} + \frac{1}{4} =$  =



Find the sum.

6. $\frac{2}{11} + \frac{3}{11}$

7. $\frac{1}{7} + \frac{5}{7}$

8. $\frac{1}{3} + \frac{1}{3}$

9. $\frac{1}{2} + \frac{1}{2}$

10. $\frac{5}{9} + \frac{1}{9}$

11. $\frac{3}{10} + \frac{5}{10}$

12. $\frac{1}{6} + \frac{5}{6}$

13. $\frac{2}{5} + \frac{1}{5}$

14. $\frac{7}{10} + \frac{9}{10}$

15. $\frac{5}{8} + \frac{7}{8}$

16. $\frac{8}{11} + \frac{2}{11}$

17. $\frac{4}{9} + \frac{2}{9}$

Find the difference.

18. $\frac{8}{9} - \frac{1}{9}$

19. $\frac{4}{5} - \frac{1}{5}$

20. $\frac{14}{17} - \frac{3}{17}$

21. $\frac{4}{6} - \frac{2}{6}$

22. $\frac{5}{8} - \frac{1}{8}$

23. $\frac{5}{6} - \frac{5}{6}$

24. $\frac{7}{9} - \frac{4}{9}$

25. $\frac{9}{14} - \frac{7}{14}$

26. $\frac{9}{10} - \frac{7}{10}$

27. $\frac{15}{16} - \frac{5}{16}$

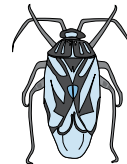
28. $\frac{11}{12} - \frac{7}{12}$

29. $\frac{14}{18} - \frac{5}{18}$

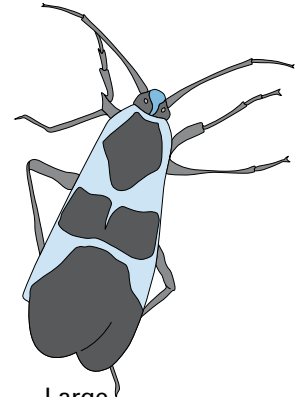
First tell whether you need to **add** or **subtract** to find the answer. Then solve the problem. Explain your reasoning.

30. You and a friend order a pizza. You eat $\frac{1}{8}$ of the pizza and your friend eats $\frac{3}{8}$ of the pizza. How much of the pizza did you and your friend eat?

31. A tarnished plant bug is about $\frac{3}{16}$ inch long. A large milkweed bug is about $\frac{9}{16}$ inch long. How much longer is the milkweed bug than the tarnished plant bug?



Tarnished plant bug

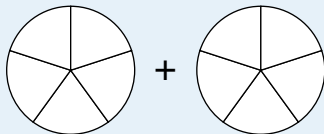


Large milkweed bug

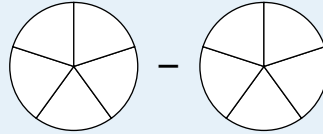
DID YOU GET IT?

32. **Fill in the missing words.** To add two fractions with like denominators, write the _____ of the _____ over the _____.
33. **Fill in the missing words.** To subtract two fractions with like denominators, write the _____ of the _____ over the _____.
34. **Use a model.** Use blank models to write (a) an addition problem and (b) a subtraction problem involving fractions. Then solve the problems.

(a)



(b)



35. **Explain your reasoning.** Your friend says that the sum of $\frac{1}{6}$ and $\frac{1}{6}$ is $\frac{2}{12}$. Is your friend correct? Explain why or why not.

LESSON 2-12



California Standards

Gr. 3 NS 3.2: Add and subtract simple fractions (e.g., determine that $\frac{1}{8} + \frac{3}{8}$ is the same as $\frac{1}{2}$).

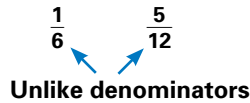
Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Also included: Gr. 3 NS 3.1

Add and Subtract Fractions with Unlike Denominators

Words to Remember

Unlike denominators: Denominators that are not equal



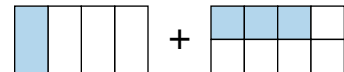
Getting Started In Lesson 2-11 you learned how to add fractions with like denominators. You can also add fractions with unlike denominators.

EXAMPLE 1 Adding Fractions Using Models

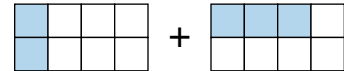
Use models to find the sum: $\frac{1}{4} + \frac{3}{8}$

Solution

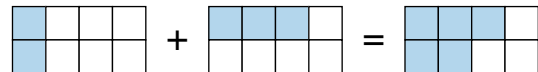
Step 1 Model the fractions.



Step 2 Redraw the models so they are divided into the same number of equal parts. Redraw $\frac{1}{4}$ as $\frac{2}{8}$ instead.



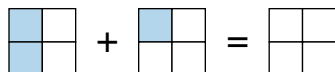
Step 3 Combine the models by shading 5 out of 8 squares.



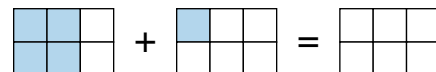
ANSWER $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

TRY THIS Use models to find the sum.

1. $\frac{1}{2} + \frac{1}{4} = \frac{\square}{4} + \frac{1}{4} = \frac{\square}{\square}$



2. $\frac{2}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} = \frac{\square}{\square}$



EXAMPLE 2**Adding Fractions with Unlike Denominators**Find the sum: $\frac{3}{5} + \frac{1}{2}$ **Solution**

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

$$+ \frac{1 \times 5}{2 \times 5} = + \frac{5}{10}$$

$$\frac{6 + 5}{10} = \frac{11}{10} = 1\frac{1}{10}$$

Use the LCD to write equivalent fractions with like denominators.

Write the sum of the numerators over the denominator.

Add and write as a mixed number.

Simplifying the Sum

If the sum is a fraction that is not in simplest form, write it as a fraction in simplest form. If the answer is an improper fraction, write it as a mixed number.

TRY THIS Find the sum.

3. $\frac{4}{9} + \frac{1}{3}$

4. $\frac{3}{4} + \frac{5}{6}$

5. $\frac{2}{7} + \frac{2}{5}$

6. $\frac{2}{3} + \frac{1}{2}$

EXAMPLE 3**Subtracting Fractions with Unlike Denominators**Find the difference: $\frac{7}{12} - \frac{1}{4}$ **Solution**

$$\frac{7}{12} = \frac{7}{12}$$

$$- \frac{1 \times 3}{4 \times 3} = - \frac{3}{12}$$

$$\frac{7 - 3}{12} = \frac{4}{12} = \frac{1}{3}$$

Use the LCD to write equivalent fractions with like denominators.

Write the difference of the numerators over the denominator.

Subtract and write the answer in simplest form.

TRY THIS Find the difference.

7. $\frac{5}{8} - \frac{1}{2}$

8. $\frac{4}{5} - \frac{1}{3}$

9. $\frac{8}{9} - \frac{5}{6}$

10. $\frac{11}{12} - \frac{1}{4}$

Summarize

Adding Fractions with Unlike Denominators

Use the LCD to write equivalent fractions with like denominators. Write the sum of the numerators over the denominator. Simplify if possible.

$$\begin{aligned}\frac{1}{3} + \frac{3}{4} &= \frac{4}{12} + \frac{9}{12} \\ &= \frac{4+9}{12} = \frac{13}{12} = 1\frac{1}{12}\end{aligned}$$

Subtracting Fractions with Unlike Denominators

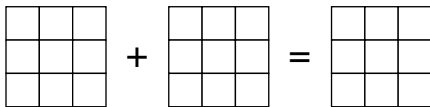
Use the LCD to write equivalent fractions with like denominators. Write the difference of the numerators over the denominator. Simplify if possible.

$$\begin{aligned}\frac{4}{5} - \frac{3}{10} &= \frac{8}{10} - \frac{3}{10} \\ &= \frac{8-3}{10} = \frac{5}{10} = \frac{1}{2}\end{aligned}$$

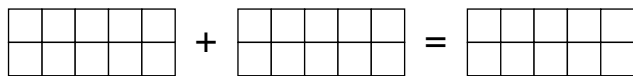
Practice

Find the sum using models.

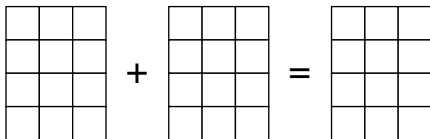
1. $\frac{2}{9} + \frac{1}{3}$



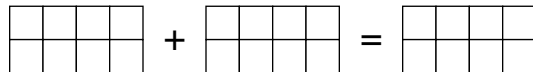
2. $\frac{3}{5} + \frac{1}{10}$



3. $\frac{1}{3} + \frac{7}{12}$



4. $\frac{3}{4} + \frac{1}{8}$



Find the sum.

5. $\frac{3}{4} + \frac{5}{8}$

6. $\frac{5}{6} + \frac{5}{12}$

7. $\frac{4}{7} + \frac{11}{14}$

8. $\frac{1}{4} + \frac{1}{6}$

9. $\frac{5}{18} + \frac{2}{9}$

10. $\frac{1}{2} + \frac{5}{8}$

11. $\frac{3}{4} + \frac{1}{5}$

12. $\frac{3}{8} + \frac{1}{2}$

Find the difference.

13. $\frac{11}{12} - \frac{1}{2}$

14. $\frac{8}{9} - \frac{2}{3}$

15. $\frac{3}{5} - \frac{1}{10}$

16. $\frac{5}{6} - \frac{3}{8}$

17. $\frac{3}{4} - \frac{3}{10}$

18. $\frac{1}{2} - \frac{1}{6}$

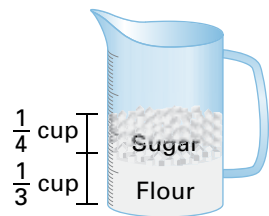
19. $\frac{4}{5} - \frac{7}{15}$

20. $\frac{7}{9} - \frac{1}{2}$

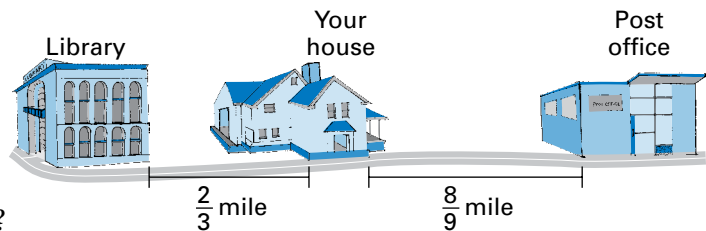
First tell whether you need to *add* or *subtract* to find the answer. Then solve the problem. Explain your reasoning.

- 21.** Kari and her grandmother are making a quilt. Over a holiday weekend, Kari sewed $\frac{1}{6}$ of the quilt. Her grandmother sewed $\frac{3}{8}$ of the quilt. How much of the quilt did they sew over the weekend?

- 22.** You put $\frac{1}{3}$ cup of flour in a 1-cup measuring cup. You then sprinkle $\frac{1}{4}$ cup of sugar on top of the flour. What fraction of the measuring cup is full?



- 23.** The distance from your house to the post office is $\frac{8}{9}$ mile. The distance from your house to the library is $\frac{2}{3}$ mile. How much farther from your house is the library than the post office?



DID YOU GET IT?

- 24. Fill in the missing words.** To add two fractions with unlike denominators, first use the _____ to write _____. Then write the _____ of the _____ over the _____.

- 25. Explain your reasoning.** Your friend says that the difference of $\frac{7}{8}$ and $\frac{1}{4}$ is $\frac{6}{4}$, or $1\frac{1}{2}$. Is your friend correct? Explain why or why not.

LESSON 2-13



California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.3: Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Add and Subtract Mixed Numbers

Getting Started In Lessons 2-11 and 2-12 you learned to add and subtract fractions. In this lesson you will add and subtract mixed numbers.

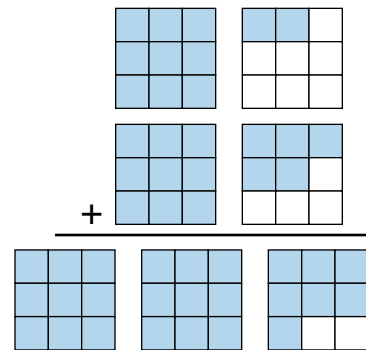
EXAMPLE 1

Adding Mixed Numbers Using Models

Use models to find the sum: $1\frac{2}{9} + 1\frac{5}{9}$

Solution

Step 1 Model the mixed numbers. Since $1 = \frac{9}{9}$, model it by shading 9 out of 9 squares.



Step 2 Combine the models.

Step 3 Count the number of shaded parts, and write this number over the denominator, 9.

$$\frac{25}{9}$$

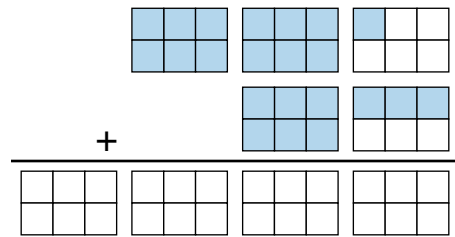
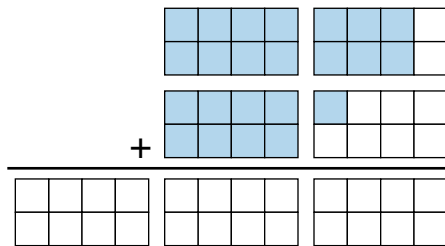
ANSWER $1\frac{2}{9} + 1\frac{5}{9} = \frac{25}{9}$ or $2\frac{7}{9}$

TRY THIS

Use models to find the sum.

1. $1\frac{6}{8} + 1\frac{1}{8} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

2. $2\frac{1}{6} + 1\frac{3}{6} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$



To Add Mixed Numbers without Using Models If the mixed numbers have the same denominator, you can add the whole numbers and fractions separately and combine the sums.

The problem in Example 1 is $1\frac{2}{9} + 1\frac{5}{9}$. When you add the whole numbers, you get $1 + 1 = 2$. When you add the fractions, you get $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$.

Combining the sums gives an answer of $2 + \frac{7}{9}$ or $2\frac{7}{9}$.

EXAMPLE 2**Adding Mixed Numbers With Unlike Denominators**Find the sum: $1\frac{2}{3} + 2\frac{3}{4}$ **Solution**

$$1\frac{2}{3} = \frac{5 \times 4}{3 \times 4} = \frac{20}{12}$$

$$+ 2\frac{3}{4} = \frac{11 \times 3}{4 \times 3} = \frac{33}{12}$$

$$= \frac{20 + 33}{12} = \frac{53}{12} = 4\frac{5}{12}$$

Write each mixed number as an improper fraction.

Use the LCD to write equivalent fractions with like denominators.

Write the sum of the numerators over the denominator and simplify.

Remember**To write a mixed number as an improper fraction:**

- Multiply the whole number by the denominator.
- Add the product to the numerator of the fraction.
- Use the denominator from the mixed number.

$$3\frac{2}{3}$$

$$3 \times 3 = 9$$

$$9 + 2 = 11$$

$$3\frac{2}{3} = \frac{11}{3}$$

TRY THIS

Find the sum.

3. $1\frac{3}{4} + 5\frac{3}{8}$

4. $2\frac{1}{2} + 1\frac{5}{6}$

5. $2\frac{1}{6} + 2\frac{4}{9}$

6. $1\frac{2}{5} + 1\frac{3}{10}$

EXAMPLE 3**Subtracting Mixed Numbers With Unlike Denominators**Find the difference: $1\frac{4}{5} - 1\frac{3}{10}$ **Solution**

$$1\frac{4}{5} = \frac{9 \times 2}{5 \times 2} = \frac{18}{10}$$

$$- 1\frac{3}{10} = \frac{13}{10} = \frac{13}{10}$$

$$= \frac{18 - 13}{10} = \frac{5}{10} = \frac{1}{2}$$

Write each mixed number as an improper fraction.

Use the LCD to write equivalent fractions with like denominators.

Write the difference of the numerators over the denominator and simplify.

TRY THIS

Find the difference.

7. $7\frac{3}{7} - 4\frac{3}{14}$

8. $3\frac{3}{5} - 3\frac{1}{4}$

9. $3\frac{2}{3} - 2\frac{2}{5}$

10. $4\frac{1}{2} - 1\frac{1}{3}$

Summarize

Adding Mixed Numbers

Write the mixed numbers as improper fractions. Use the LCD to write equivalent fractions with like denominators. Write the sum of the numerators over the denominator and simplify.

$$\begin{array}{r} 1\frac{1}{4} = \frac{5}{4} = \frac{5}{4} \\ + 1\frac{1}{2} = \frac{3 \times 2}{2 \times 2} = + \frac{6}{4} \\ \hline = \frac{5+6}{4} = \frac{11}{4} = 2\frac{3}{4} \end{array}$$

Subtracting Mixed Numbers

Write the mixed numbers as improper fractions. Use the LCD to write equivalent fractions with like denominators. Write the difference of the numerators over the denominator and simplify.

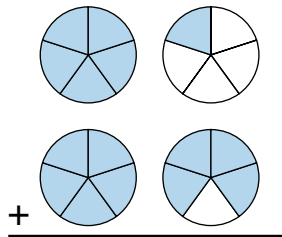
$$\begin{array}{r} 1\frac{5}{6} = \frac{11 \times 2}{6 \times 2} = \frac{22}{12} \\ - 1\frac{1}{12} = \frac{13}{12} = - \frac{13}{12} \\ \hline = \frac{22-13}{12} = \frac{9}{12} = \frac{3}{4} \end{array}$$

Practice

Match the addition problem with the model that represents it. Then write the sum.

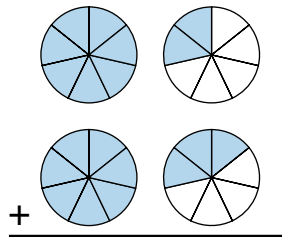
1. $1\frac{3}{8} + 1\frac{1}{8}$ _____

A.



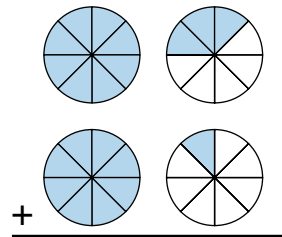
2. $1\frac{1}{5} + 1\frac{4}{5}$ _____

B.



3. $1\frac{2}{7} + 1\frac{3}{7}$ _____

C.



Find the sum.

4. $2\frac{2}{5} + 1\frac{8}{15}$

5. $1\frac{2}{3} + 2\frac{1}{9}$

6. $1\frac{11}{20} + 3\frac{7}{10}$

7. $3\frac{1}{4} + 2\frac{1}{6}$

8. $5\frac{1}{2} + 1\frac{5}{8}$

9. $1\frac{5}{6} + 2\frac{2}{9}$

10. $4\frac{1}{2} + 1\frac{7}{9}$

11. $2\frac{3}{4} + 1\frac{3}{5}$

12. $2\frac{1}{6} + 1\frac{7}{18}$

Find the difference.

13. $2\frac{1}{2} - 1\frac{1}{6}$

14. $3\frac{4}{7} - 2\frac{1}{7}$

15. $5\frac{1}{4} - 2\frac{3}{10}$

16. $3\frac{1}{8} - 1\frac{11}{16}$

17. $4\frac{9}{10} - 3\frac{1}{2}$

18. $3\frac{3}{4} - 2\frac{1}{6}$

19. $1\frac{5}{7} - 1\frac{1}{2}$

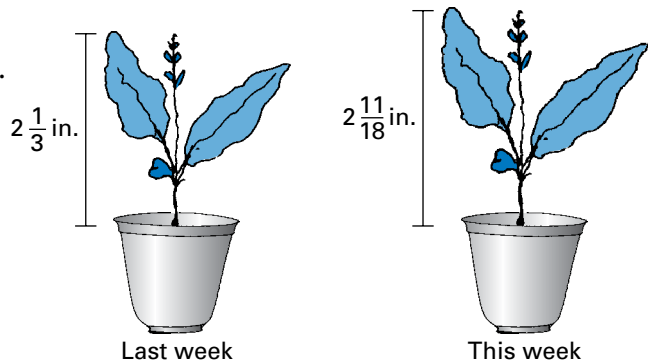
20. $2\frac{2}{3} - 1\frac{11}{12}$

21. $3\frac{4}{5} - 1\frac{1}{4}$

First tell whether you need to *add* or *subtract* to find the answer. Then solve the problem. Explain your reasoning.

- 22.** Your mom buys $2\frac{1}{2}$ yards of material. She returns to the store the next day and buys $1\frac{3}{4}$ yards of material. How much material does she buy in all?

- 23.** Last week your plant measured $2\frac{1}{3}$ inches tall. This week it measures $2\frac{11}{18}$ inches tall. How much has your plant grown?



- 24.** A red carpet pad is $2\frac{5}{6}$ centimeters thick. A blue carpet pad is $1\frac{1}{4}$ centimeters thinner than the red pad. How thick is the blue pad?

DID YOU GET IT?

- 25. Fill in the missing words.** To subtract two _____, first write the _____ as _____. Then write the _____ of the _____ over the _____.

- 26. Describe a process.** How can you rewrite the mixed numbers $4\frac{1}{5}$ and $2\frac{7}{10}$ so that you can add them?

LESSON
2-14

California
Standards

Gr. 5 NS 2.5: Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Gr. 6 NS 2.2: Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g.,

$$\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}.$$

Also included: Gr. 5 NS 2.0

Multiply Fractions

Words to Remember

$$\frac{2}{3} \text{ of } 4 \text{ means } \frac{2}{3} \times 4$$

$$\frac{1}{4} \text{ of } \frac{1}{2} \text{ means } \frac{1}{4} \times \frac{1}{2}$$

Getting Started You multiplied two whole numbers. You can also multiply a whole number and a fraction or two fractions.

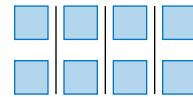
EXAMPLE 1

Multiplying a Fraction By a Whole Number Using Models

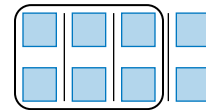
Use models to find the product: $\frac{3}{4}$ of 8 or $\frac{3}{4} \times 8$

Solution

Step 1 Draw 8 squares. Arrange them in two rows of four. Notice that there are 4 columns and there are 2 squares in each column.



Step 2 Draw a circle around 3 of the 4 columns of squares. This represents $\frac{3}{4}$ of 8, or $\frac{3}{4} \times 8$. Notice that you have circled 6 squares.



ANSWER $\frac{3}{4} \times 8 = 6$

TRY THIS

Use models to find the product.

1. $\frac{1}{3} \times 9$

2. $\frac{2}{5}$ of 10

Multiplying a Fraction by a Whole Number To multiply a fraction by a whole number, write the product of the numerator and the whole number over the denominator. Then simplify if possible.

$$\frac{3}{4} \text{ of } 6 = \frac{3}{4} \times 6 = \frac{3 \times 6}{4} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}$$

EXAMPLE 2**Multiplying a Fraction by a Whole Number**Find the product: $14 \times \frac{1}{6}$ **Solution**

Write the product of the whole number and the numerator over the denominator and simplify.

$$14 \times \frac{1}{6} = \frac{14 \times 1}{6} = \frac{14}{6} = \frac{7}{3} = 2\frac{1}{3}$$

TRY THIS

Find the product.

$$3. \quad 15 \times \frac{3}{5} = \frac{\square \times \square}{\square} = \frac{\square}{\square} = \square$$

$$4. \quad \frac{5}{6} \text{ of } 12 = \frac{\square \times \square}{\square} = \frac{\square}{\square} = \square$$

$$5. \quad \frac{2}{7} \times 21$$

$$6. \quad \frac{2}{9} \text{ of } 5$$

Another Way

You can also simplify a fraction before multiplying.

In Try this 3,

$$\frac{15 \times 3}{5} = \frac{\overset{3}{\cancel{15}} \times 3}{\underset{1}{\cancel{5}}} = \frac{9}{1} = 9$$

Multiplying Two Fractions To multiply two fractions, write the product of the numerators over the product of the denominators. Then simplify if possible.

$$\frac{1}{9} \times \frac{3}{5} = \frac{1 \times 3}{9 \times 5} = \frac{3}{45} = \frac{1}{15}$$

EXAMPLE 3**Multiplying a Fraction by a Fraction**Find the product: $\frac{2}{3} \times \frac{3}{4}$ **Solution**

Write the product of the numerators over the product of the denominators and simplify.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

Watch Out!

Don't forget to multiply the denominators, too.

TRY THIS

Find the product.

$$7. \quad \frac{3}{4} \text{ of } \frac{2}{5} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square} = \frac{\square}{\square}$$

$$8. \quad \frac{1}{6} \times \frac{3}{5} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square} = \frac{\square}{\square}$$

$$9. \quad \frac{5}{8} \times \frac{4}{9}$$

$$10. \quad \frac{1}{4} \text{ of } \frac{2}{7}$$

Summarize**Multiplying a Fraction by a Whole Number**

Write the product of the whole number and the numerator over the denominator. Simplify if possible.

$$2 \times \frac{4}{9} = \frac{2 \times 4}{9} = \frac{8}{9}$$

Multiplying Two Fractions

Write the product of the numerators over the product of the denominators. Simplify if possible.

$$\frac{3}{8} \times \frac{2}{9} = \frac{3 \times 2}{8 \times 9} = \frac{6}{72} = \frac{1}{12}$$

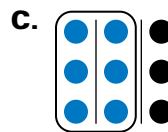
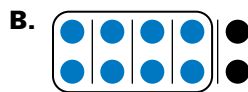
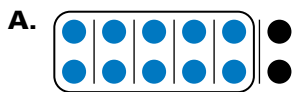
Practice

Match the multiplication problem with the model that represents it.

1. $\frac{4}{5}$ of 10 _____

2. $\frac{2}{3} \times 9$ _____

3. $\frac{5}{6} \times 12$ _____

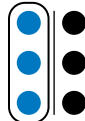
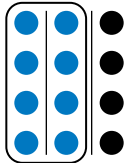


Write the multiplication problem that the model represents.

4. _____

5. _____

6. _____



Find the product of the fraction and whole number.

7. $\frac{4}{5} \times 20$

8. $\frac{5}{8}$ of 16

9. $\frac{3}{4} \times 16$

10. $\frac{1}{3} \times 14$

11. $\frac{2}{7}$ of 7

12. $\frac{5}{6}$ of 10

13. $\frac{1}{2}$ of 5

14. $\frac{4}{9} \times 21$

15. $\frac{3}{8} \times 4$

Find the product of the fractions.

16. $\frac{3}{4} \times \frac{4}{9}$

17. $\frac{1}{2}$ of $\frac{2}{5}$

18. $\frac{1}{6}$ of $\frac{9}{10}$

19. $\frac{5}{9}$ of $\frac{3}{7}$

20. $\frac{2}{3} \times \frac{6}{11}$

21. $\frac{3}{5}$ of $\frac{1}{3}$

22. $\frac{4}{11} \times \frac{1}{2}$

23. $\frac{5}{8} \times \frac{4}{7}$

24. $\frac{7}{10} \times \frac{5}{14}$

Solve the problem. Explain your reasoning.

- 25.** Two-thirds of **21** students on a field trip brought a bag lunch. How many students brought a bag lunch?

- 26.** There is a $\frac{3}{4}$ -mile walking trail around the lake at your park. You walk around this trail **5** times. What is the total distance that you walked?

- 27.** Your mom says she spent $\frac{1}{4}$ of $\frac{1}{2}$ hour reading through the mail. How much time did she spend reading through the mail?

DID YOU GET IT?

- 28. Fill in the missing words.** To multiply a fraction by a fraction write the _____ of the _____ over the _____ of the _____.

- 29. Use a model.** Write and solve a multiplication problem using the blank model at the right.

- 30. Write a problem.** Write a multiplication problem with two different fractions so that when they are multiplied together, you get the same answer as in the problem at the right.

$$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$$

$$\square \times \square = \frac{1}{10}$$

LESSON
2-15

California
Standards

Gr. 6 NS 2.2: Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g.

$$\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}.$$

Algebra 1 2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

Also included: Gr. 5 NS 2.0, Gr. 5 NS 2.5, Gr. 7 AF 1.3

Divide Fractions

Words to Remember

Reciprocals: Two numbers whose product is 1

$\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals because

$$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$$

5 and $\frac{1}{5}$ are reciprocals because

$$5 \times \frac{1}{5} = \frac{5 \times 1}{5} = \frac{5}{5} = 1$$

Inverse property of multiplication:
The product of a number and its reciprocal is 1.

$$a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$$

Getting Started In Lesson 2-14 you learned how to multiply fractions. You can also divide fractions by using a reciprocal.

EXAMPLE 1

Finding the Reciprocal of a Number

Find the reciprocal of the number.

a. $\frac{1}{8}$

b. 4

Solution

- a. If the number is a fraction, write the denominator over the numerator.

ANSWER $\frac{8}{1}$ or 8

- b. If the number is a whole number, write it as a fraction. Then write the denominator over the

numerator. $4 = \frac{4}{1}$

ANSWER $\frac{1}{4}$

TRY THIS

Find the reciprocal of the number.

1. $\frac{7}{11}$

2. $\frac{1}{15}$

3. 3

Dividing Fractions To divide two fractions, rewrite the problem as a multiplication problem. In the new problem, multiply the dividend by the reciprocal of the divisor. The dividend is the first number in a division problem. The divisor is the second number.

Dividing Fractions (*cont.*) When dividing two fractions, the first fraction stays the same. The second fraction is rewritten as a reciprocal. The denominator becomes the numerator and the numerator becomes the denominator. The operation changes from division to multiplication.

$$\begin{array}{ccccccc} \frac{1}{4} & \div & \frac{5}{6} & = & \frac{1}{4} & \times & \frac{6}{5} \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{Dividend} & & \text{Divisor} & & \text{Dividend} & & \text{Reciprocal of the Divisor} \end{array}$$

EXAMPLE 2**Dividing a Fraction by a Fraction**

Divide: $\frac{2}{5} \div \frac{4}{9}$

Solution

$$\begin{aligned} \frac{2}{5} \div \frac{4}{9} &= \frac{2}{5} \times \frac{9}{4} \\ &= \frac{2 \times 9}{5 \times 4} \\ &= \frac{18}{20} = \frac{9}{10} \end{aligned}$$

Rewrite the problem so that the dividend is multiplied by the reciprocal of the divisor.

Multiply the fractions.

Simplify the answer, if possible.

TRY THIS Divide.

4. $\frac{2}{3} \div \frac{1}{3}$

5. $\frac{1}{4} \div \frac{4}{9}$

6. $\frac{3}{5} \div \frac{7}{10}$

EXAMPLE 3**Dividing Fractions and Whole Numbers**

Divide.

a. $\frac{3}{8} \div 6$

b. $4 \div \frac{1}{6}$

Solution

$$\begin{aligned} \text{a. } \frac{3}{8} \div \frac{6}{1} & \\ &= \frac{3}{8} \times \frac{1}{6} \\ &= \frac{3 \times 1}{8 \times 6} \\ &= \frac{3}{48} = \frac{1}{16} \end{aligned}$$

Write the whole number as a fraction.

Rewrite the problem so that the dividend is multiplied by the reciprocal of the divisor.

Multiply the fractions.

Simplify the answer, if possible.

$$\begin{aligned} \text{b. } 4 \div \frac{1}{6} & \\ &= \frac{4}{1} \times \frac{6}{1} \\ &= \frac{4 \times 6}{1 \times 1} \\ &= \frac{24}{1} = 24 \end{aligned}$$

Check Your Answer

Multiply your answer by the divisor to see if you get the dividend.

In Example 3 part (a), $\frac{1}{16} \times 6 = \frac{6}{16} = \frac{3}{8}$. The dividend is $\frac{3}{8}$, so the answer checks.

TRY THIS Divide.

7. $\frac{6}{11} \div 3$

8. $\frac{5}{8} \div 5$

9. $8 \div \frac{2}{3}$

Summarize

Finding the Reciprocal

Make sure the number is written as a fraction. Then write the denominator over the numerator and simplify.

$$\text{Reciprocal of } \frac{2}{11} : \frac{11}{2} = 5\frac{1}{2}$$

$$\text{Reciprocal of } 3 \text{ or } \frac{3}{1} : \frac{1}{3}$$

Dividing Two Fractions

Rewrite the problem so that the dividend is multiplied by the reciprocal of the divisor. Multiply the fractions. Then, if possible, simplify your answer.

$$\frac{4}{5} \div \frac{5}{6} = \frac{4}{5} \times \frac{6}{5} = \frac{24}{25}$$

Dividing Whole Numbers and Fractions

Write the whole number as a fraction. Rewrite the problem so that the dividend is multiplied by the reciprocal of the divisor. Multiply and then simplify, if possible.

$$3 \div \frac{4}{9} = \frac{3}{1} \times \frac{9}{4} = \frac{27}{4} \text{ or } 6\frac{3}{4}$$

$$\frac{1}{2} \div 4 = \frac{1}{2} \div \frac{4}{1} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Practice

Find the reciprocal of the number.

1. $\frac{2}{7}$

2. $\frac{3}{4}$

3. $\frac{5}{12}$

4. $\frac{1}{11}$

5. $\frac{1}{2}$

6. 9

7. 15

8. 3

Match the division problem with the multiplication problem that represents it.

9. $\frac{3}{5} \div \frac{1}{4}$ _____

A. $\frac{3}{5} \times \frac{1}{4}$

10. $\frac{3}{5} \div 4$ _____

B. $\frac{1}{4} \times \frac{5}{3}$

11. $4 \div \frac{3}{5}$ _____

C. $\frac{3}{5} \times \frac{4}{1}$

12. $\frac{1}{4} \div \frac{3}{5}$ _____

D. $\frac{4}{1} \times \frac{5}{3}$

Divide.

13. $\frac{3}{7} \div \frac{4}{7}$

14. $\frac{5}{9} \div \frac{1}{3}$

15. $\frac{1}{10} \div \frac{2}{11}$

16. $\frac{4}{5} \div 3$

17. $\frac{7}{12} \div 2$

18. $\frac{2}{3} \div 8$

19. $5 \div \frac{3}{4}$

20. $6 \div \frac{1}{5}$

21. $9 \div \frac{6}{7}$

22. $\frac{7}{8} \div \frac{2}{3}$

23. $\frac{2}{5} \div 5$

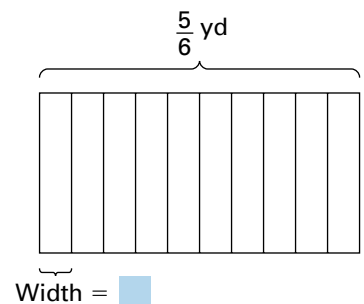
24. $4 \div \frac{5}{8}$

Solve the problem. Explain your reasoning.

- 25.** You use $\frac{2}{3}$ cup of sugar when you make a batch of brownies. How many batches could you make with **8** cups of sugar?

- 26.** How many $\frac{1}{8}$ -gallon servings could you pour from $\frac{3}{4}$ gallon of milk?

- 27.** You have a poster board that is $\frac{5}{6}$ yard wide. You want to divide the poster into **10** equal sections. Find the width of one of these sections.



DID YOU GET IT?

- 28. Fill in the missing words.** To find the reciprocal of a fraction write the _____ over the _____.

- 29. Explain your reasoning.** Malik says that $\frac{8}{15} \div \frac{5}{6}$ is equal to $\frac{8}{15} \times \frac{5}{6}$, or $\frac{4}{9}$. Is he correct? Why or why not?

- 30. Write a problem.** Write a division problem that could be rewritten as the multiplication problem $\frac{4}{5} \times \frac{1}{4}$.

LESSON
2-16

California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.5: Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Also included: Gr. 6 NS 2.2

Multiply and Divide Mixed Numbers

Getting Started In Lessons 2-14 and 2-15 you learned how to multiply and divide fractions. You can also multiply and divide mixed numbers.

Multiplying and Dividing Mixed Numbers When you multiply or divide a mixed number, the first step is always to rewrite the mixed number as an improper fraction.

EXAMPLE 1
Multiplying Mixed Numbers and Fractions

Multiply.

a. $2\frac{2}{5} \times \frac{1}{4}$

b. $\frac{3}{10} \times 1\frac{2}{3}$

Solution

Write the mixed numbers as improper fractions and multiply.

$$\begin{aligned} \text{a. } 2\frac{2}{5} \times \frac{1}{4} &= \frac{(2 \times 5) + 2}{5} \times \frac{1}{4} \\ &= \frac{12}{5} \times \frac{1}{4} \\ &= \frac{12 \times 1}{5 \times 4} \\ &= \frac{12}{20} = \frac{3}{5} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{3}{10} \times 1\frac{2}{3} &= \frac{3}{10} \times \frac{(1 \times 3) + 2}{3} \\ &= \frac{3}{10} \times \frac{5}{3} \\ &= \frac{3 \times 5}{10 \times 3} \\ &= \frac{15}{30} = \frac{1}{2} \end{aligned}$$

EXAMPLE 2
Multiplying Mixed Numbers and Whole Numbers

Multiply.

a. $1\frac{4}{9} \times 3$

b. $2 \times 1\frac{5}{6}$

Solution

Write the whole numbers as improper fractions and multiply as in Example 1.

$$\begin{aligned} \text{a. } 1\frac{4}{9} \times 3 &= \frac{13}{9} \times \frac{3}{1} \\ &= \frac{13 \times 3}{9 \times 1} \\ &= \frac{39}{9} = 4\frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{b. } 2 \times 1\frac{5}{6} &= \frac{2}{1} \times \frac{11}{6} \\ &= \frac{2 \times 11}{1 \times 6} \\ &= \frac{22}{6} = 3\frac{2}{3} \end{aligned}$$

Watch Out!

You also need to write the whole number as an improper fraction. Write the number over a denominator of 1.

EXAMPLE 3**Dividing Mixed Numbers and Fractions**

Divide.

a. $1\frac{3}{4} \div \frac{5}{12}$

b. $\frac{7}{8} \div 2\frac{5}{8}$

Remember

When you divide by a fraction, you must multiply by the reciprocal of the divisor. To write a reciprocal, reverse the numerator and the denominator.

Solution

$$\begin{aligned} \text{a. } 1\frac{3}{4} \div \frac{5}{12} &= \frac{7}{4} \div \frac{5}{12} \\ &= \frac{7}{4} \times \frac{12}{5} \\ &= \frac{7 \times 12}{4 \times 5} \\ &= \frac{84}{20} = 4\frac{1}{5} \end{aligned}$$

Write as an improper fraction.

Multiply by the reciprocal of the divisor.

Multiply.

Simplify.

$$\begin{aligned} \text{b. } \frac{7}{8} \div 2\frac{5}{8} &= \frac{7}{8} \div \frac{21}{8} \\ &= \frac{7}{8} \times \frac{8}{21} \\ &= \frac{7 \times 8}{8 \times 21} \\ &= \frac{56}{168} = \frac{1}{3} \end{aligned}$$

Write as an improper fraction.

Multiply by the reciprocal of the divisor.

Multiply.

Simplify.

EXAMPLE 4**Dividing Mixed Numbers and Whole Numbers**

Divide.

a. $1\frac{4}{7} \div 4$

b. $6 \div 4\frac{1}{2}$

Solution

Write whole numbers as improper fractions. Then divide as in Example 3.

$$\begin{aligned} \text{a. } 1\frac{4}{7} \div 4 &= \frac{11}{7} \div \frac{4}{1} \\ &= \frac{11}{7} \times \frac{1}{4} \\ &= \frac{11 \times 1}{7 \times 4} \\ &= \frac{11}{28} \end{aligned}$$

$$\begin{aligned} \text{b. } 6 \div 4\frac{1}{2} &= \frac{6}{1} \div \frac{9}{2} \\ &= \frac{6}{1} \times \frac{2}{9} \\ &= \frac{6 \times 2}{1 \times 9} \\ &= \frac{12}{9} = 1\frac{1}{3} \end{aligned}$$

TRY THIS

Multiply or divide.

1. $2\frac{1}{6} \times \frac{1}{3}$

2. $\frac{2}{5} \times 3\frac{1}{2}$

3. $2 \times 1\frac{7}{12}$

4. $2\frac{5}{8} \times 3$

5. $2\frac{2}{3} \div \frac{3}{4}$

6. $\frac{1}{2} \div 1\frac{5}{6}$

7. $4 \div 1\frac{1}{7}$

8. $3\frac{1}{8} \div 5$

Summarize**Multiplying Mixed Numbers and Fractions**

Write the mixed number as an improper fraction.
Multiply and simplify.

$$\begin{aligned} 1\frac{1}{2} \times \frac{4}{7} &= \frac{3}{2} \times \frac{4}{7} \\ &= \frac{3 \times 4}{2 \times 7} = \frac{12}{14} = \frac{6}{7} \end{aligned}$$

Multiplying Mixed Numbers and Whole Numbers

Write the mixed number and whole number as improper fractions. Multiply and simplify.

$$\begin{aligned} 8 \times 1\frac{3}{8} &= \frac{8}{1} \times \frac{11}{8} \\ &= \frac{8 \times 11}{1 \times 8} = \frac{88}{8} = 11 \end{aligned}$$

Dividing Mixed Numbers and Fractions

Write the mixed number as an improper fraction.
Multiply the dividend by the reciprocal of the divisor and simplify.

$$\begin{aligned} \frac{5}{9} \div 1\frac{1}{3} &= \frac{5}{9} \div \frac{4}{3} \\ &= \frac{5}{9} \times \frac{3}{4} \\ &= \frac{5 \times 3}{9 \times 4} = \frac{15}{36} = \frac{5}{12} \end{aligned}$$

Dividing Mixed Numbers and Whole Numbers

Write the mixed number and whole number as improper fractions. Multiply the dividend by the reciprocal of the divisor and simplify.

$$\begin{aligned} 1\frac{2}{3} \div 3 &= \frac{5}{3} \div \frac{3}{1} \\ &= \frac{5}{3} \times \frac{1}{3} \\ &= \frac{5 \times 1}{3 \times 3} = \frac{5}{9} \end{aligned}$$

Practice

Match the problem with a problem that is the same.

1. $4\frac{2}{5} \div 8$ _____

2. $8 \times 4\frac{2}{5}$ _____

3. $\frac{1}{8} \div 1\frac{3}{5}$ _____

4. $1\frac{3}{5} \times \frac{1}{8}$ _____

A. $\frac{22}{5} \times \frac{1}{8}$

B. $\frac{8}{5} \times \frac{1}{8}$

C. $8 \times \frac{22}{5}$

D. $\frac{1}{8} \times \frac{5}{8}$

Multiply.

5. $2\frac{3}{4} \times \frac{1}{7}$

6. $4\frac{1}{4} \times \frac{4}{5}$

7. $\frac{1}{6} \times 3\frac{3}{7}$

8. $\frac{9}{10} \times 1\frac{2}{3}$

9. $1\frac{5}{6} \times 4$

10. $3\frac{5}{8} \times 2$

11. $5 \times 1\frac{3}{4}$

12. $3 \times 2\frac{7}{12}$

13. $2\frac{1}{4} \times \frac{2}{3}$

14. $2 \times 4\frac{7}{10}$

15. $2\frac{2}{3} \times 5$

16. $\frac{3}{7} \times 2\frac{1}{3}$

Divide.

17. $1\frac{9}{10} \div \frac{2}{5}$

18. $1\frac{5}{12} \div \frac{1}{6}$

19. $\frac{7}{8} \div 2\frac{1}{2}$

20. $\frac{1}{6} \div 1\frac{1}{4}$

21. $3\frac{1}{5} \div 2$

22. $4\frac{4}{9} \div 4$

23. $5 \div 1\frac{1}{6}$

24. $9 \div 6\frac{2}{3}$

25. $2\frac{1}{3} \div \frac{2}{5}$

26. $8 \div 3\frac{1}{5}$

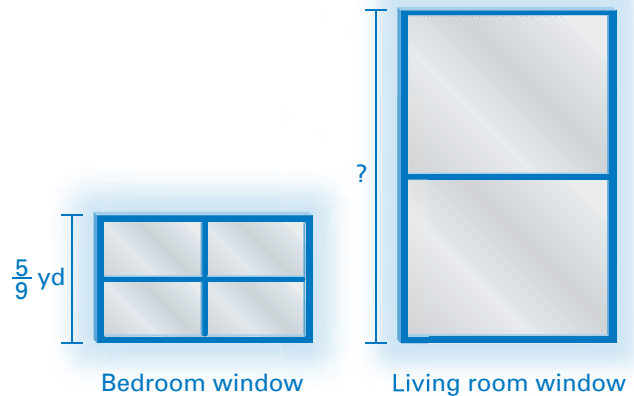
27. $\frac{4}{9} \div 1\frac{1}{9}$

28. $4\frac{2}{3} \div 3$

First tell whether you will *multiply* or *divide*. Then solve the problem. Explain your reasoning.

- 29.** You are making 5 birdhouses. You need $1\frac{1}{4}$ quarts of paint to paint 1 birdhouse. How many quarts of paint will you need in all?

- 30.** Your bedroom window is $\frac{5}{9}$ yard tall. Your living room window is $2\frac{3}{5}$ times as tall as your bedroom window. How tall is your living room window?



- 31.** You have 12 square feet of land that you are going to separate into sections of $2\frac{2}{5}$ square feet each. How many sections will you have?

DID YOU GET IT?

- 32. Fill in the missing words.** To multiply a mixed number by a fraction, first write the _____ as a(n) _____.

- 33. Explain your reasoning.** Becca says that $7 \div 1\frac{2}{7}$ is the same as $\frac{7}{1} \times \frac{9}{7}$. Is she correct? Why or why not?

LESSON 2-17



California Standards

Gr. 5 NS 2.5: Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

Gr. 6 NS 2.1: Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

Also included: Gr. 5 NS 2.3, Gr. K/1/2 MR 2.2, Gr. 3/4/5 MR 2.6, Gr. 6 MR 2.7, and Gr. 7 MR 2.8

Solve Problems with Fractions

Strategies to Remember

To decide which information in a word problem is important and to order the steps you will follow:

Identify the <i>given information</i>: <ul style="list-style-type: none"> • dollar amounts • number of items • number of groups • weights, masses, lengths, etc. 	Identify the <i>operation(s)</i> needed: <ul style="list-style-type: none"> • addition • subtraction • multiplication • division
Identify the <i>calculations</i> you must make to solve the problem: <ul style="list-style-type: none"> • decide which pieces of information you need to use with each operation 	Order the steps you will follow to solve the problem: <ul style="list-style-type: none"> • decide on an order for the calculations

EXAMPLE 1

Solving an Addition Problem

A school's rain gauge catches $\frac{3}{8}$ inch of rain on Saturday and $\frac{1}{2}$ inch of rain on Sunday. What is the total rainfall for the weekend?

Solution

Step 1 Identify the operation needed. The word “total” tells you to *add*.

Step 2 Add the fractions: $\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$

Step 3 Check for reasonableness. Because $\frac{3}{8}$ is almost $\frac{1}{2}$ and $\frac{1}{2} + \frac{1}{2} = 1$, the answer should be slightly less than 1. So, $\frac{7}{8}$ inch is reasonable.

ANSWER The total rainfall for the weekend is $\frac{7}{8}$ inch.

TRY THIS

Fill in the missing information to solve the problem.

1. A package of 1 lb of hamburger meat loses $\frac{1}{4}$ lb when it is cooked. How much does the cooked hamburger meat weigh?

Step 1 Identify the operation needed. The word “loses” tells you to ____.

Step 2 Calculate: $1 \text{ } \bigcirc \text{ } \frac{1}{4} = \frac{\text{ } }{\text{ } }$

Step 3 Check for reasonableness. Because $\frac{1}{4}$ is close to ____ and $1 - \text{ } = \text{ }$, the answer should be close to ____.

Step 4 The cooked hamburger meat weighs _____ lb.

EXAMPLE 2**Solving a Division Problem**

You are making dog leashes from a 50-foot length of rope. It takes $7\frac{1}{2}$ ft of rope to make each leash. How many leashes can you make?

Solution

Step 1 Identify the operation needed. To find how many $7\frac{1}{2}$ -foot leashes can be made from 50 feet of rope, you need to *divide*.

Step 2 Calculate. Divide 50 by $7\frac{1}{2}$:

$$\begin{aligned} 50 \div 7\frac{1}{2} &= \frac{50}{1} \div \frac{15}{2} \\ &= \frac{50}{1} \times \frac{2}{15} \\ &= \frac{100}{15} = \frac{20}{3} = 6\frac{2}{3} \end{aligned}$$

Step 3 Check that $6\frac{2}{3}$ leashes is reasonable. It does not make sense to make $\frac{2}{3}$ of a leash. So, 6 leashes is a more reasonable answer.

ANSWER You can make 6 leashes.

EXAMPLE 3**Solving Problems Using Multiple Operations**

You fill $\frac{1}{2}$ -liter bottles from a cooler containing $11\frac{1}{4}$ L of water. How much water will be left in the cooler after filling 21 bottles?

Solution

Solve the problem. Identify the operations. Decide the order for the operations, then calculate. Check that your answer is reasonable.

Multiply, then subtract. $\frac{1}{2} \times 21 = \frac{1}{2} \times \frac{21}{1} = \frac{21}{2}$ or $10\frac{1}{2}$

$$\begin{aligned} 11\frac{1}{4} - \frac{21}{2} &= \frac{45}{4} - \frac{21}{2} \\ &= \frac{45}{4} - \frac{42}{4} = \frac{3}{4} \end{aligned}$$

CHECK $11\frac{1}{4}$ is close to 11 and $11 - 10\frac{1}{2} = \frac{1}{2}$. So, $\frac{3}{4}$ L is reasonable.

TRY THIS

Solve the problem. Explain your reasoning.

2. You have a 30-foot roll of art paper. If an art project takes $1\frac{1}{3}$ feet of art paper, how much paper will be left over after you make 12 projects?

Summarize**Solving Word Problems**

- (1) Identify the operation or operations needed.
- (2) Perform the calculations and solve the problem.
- (3) Check your answer for reasonableness.

Practice

Fill in the missing operation symbol $+$, $-$, \times , or \div for the situation described.

1. A student received $\frac{1}{5}$ of the class's 120 votes. $\frac{1}{5} \bigcirc 120$
2. A cat weighs $1\frac{1}{2}$ pounds more than its usual weight of 9 pounds.
 $9 \bigcirc 1\frac{1}{2}$
3. Bracelets that are $6\frac{1}{4}$ inches long are being made from 72 inches of wire. $72 \bigcirc 6\frac{1}{4}$
4. You have $\frac{1}{3}$ cup less flour than the $2\frac{1}{2}$ cups the recipe suggests. $2\frac{1}{2} \bigcirc \frac{1}{3}$

Identify the operation suggested by the phrase.

5. took away $\frac{1}{4}$ pound
6. $\frac{2}{3}$ foot and $\frac{1}{2}$ foot
7. half of the 30 students

Read the problem. The first two steps of the solution are given. Number the steps of the solution to put them in order.

8. A kitten weighs $\frac{7}{8}$ pound. The kitten gains $\frac{1}{4}$ pound the first week and $\frac{1}{8}$ pound the second week. How much does the kitten weigh after 2 weeks?

SOLUTION: ____ add: $1\frac{1}{8} + \frac{1}{8}$ ____ add: $\frac{7}{8} + \frac{1}{4}$

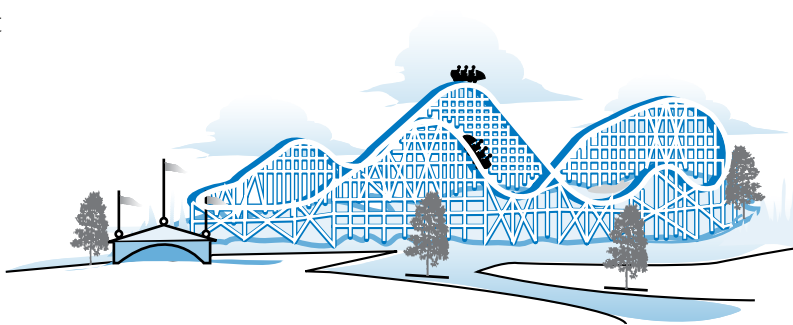
9. You need 4 pounds of pears for a dessert recipe. Each pear weighs about $\frac{2}{5}$ pound. After putting several pears on the scale in a grocery store, you still need $1\frac{3}{5}$ pounds of pears. How many pears are in the scale right now?

SOLUTION: ____ subtract: $4 - 1\frac{3}{5}$ ____ divide: $2\frac{2}{5} \div \frac{2}{5}$

First identify the operation(s) needed to find the answer. Then solve the problem. Check your answer for reasonableness. Explain your reasoning.

- 10.** Mario saves $\frac{1}{5}$ of his summer earnings for college. Last summer, Mario earned \$2560. How much of this amount did he save for college?

- 11.** The wait time at an amusement park ride is $1\frac{1}{2}$ hours. You have already been waiting in line for $\frac{3}{4}$ hour. How many minutes do you have left to wait?
Hint: There are 60 minutes in 1 hour.



- 12.** On Julio's computer, applications take up $\frac{9}{80}$ of the hard drive space and all other files take up $\frac{1}{20}$ of the hard drive space. What fraction of the hard drive space is left?

DID YOU GET IT?

- 13. Write a word problem.** Write a word problem using the fractions at the right. Then solve the problem. Explain how you decided which operation to use.

$$\frac{3}{8} \bigcirc \frac{1}{4} = \square$$

- 14. Explain your reasoning.** Your friend says that the answer to the following problem is 4 shelves. Is your friend correct? Explain why or why not. A carpenter is cutting a 7-foot-long board to make shelves for a bookcase. If the carpenter wants each shelf to be $\frac{15}{6}$ feet long, how many shelves can be made?

Mixed Practice for Lessons 2-11 to 2-17

Vocabulary Review

Match the word with its definition.

Word

Definition

- | | | |
|----------------|-------|---|
| 1. operation | _____ | A. two fractions whose product is 1 |
| 2. difference | _____ | B. an action you perform on numbers, such as addition or multiplication |
| 3. reciprocals | _____ | C. the answer when you subtract one number from another |

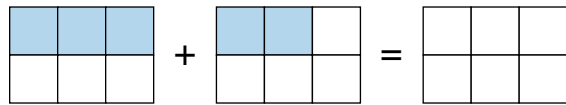
Fill in the missing word(s).

4. After you solve a word problem, it is important to check for _____.
5. To subtract two fractions with unlike denominators, use the _____
_____ to write the fractions with like denominators.

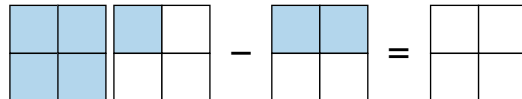
Shade the models below to find the sum or difference of the fractions.

6. $\frac{4}{7} - \frac{3}{7} = \frac{\square}{\square}$

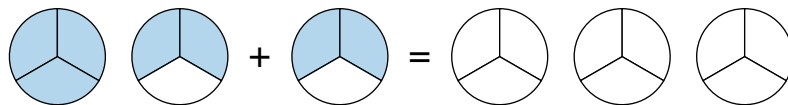
7. $\frac{3}{6} + \frac{2}{6} = \frac{\square}{\square}$



8. $1\frac{1}{4} - \frac{2}{4} = \frac{\square}{\square}$



9. $1\frac{2}{3} + \frac{2}{3} = \frac{\square}{\square}$



Add, subtract, multiply, or divide. Write your answer in simplest form.

10. $\frac{2}{7} + \frac{3}{7}$

11. $\frac{2}{9} \times \frac{1}{3}$

12. $\frac{3}{4} \div \frac{8}{3}$

13. $\frac{7}{8} - \frac{2}{8}$

14. $\frac{5}{6} \div 7$

15. $6\frac{5}{9} - 1\frac{4}{9}$

16. $20 \times \frac{1}{4}$

17. $\frac{3}{4} + \frac{3}{4}$

18. $\frac{3}{10} + 3\frac{1}{10}$

19. $\frac{9}{10} \times \frac{2}{3}$

20. $\frac{4}{5} \div \frac{2}{5}$

21. $7\frac{5}{6} - 5\frac{1}{6}$

22. Fill in the missing information to solve the problem.

Students at a school are performing in a play. At the play, adults make up $\frac{2}{5}$ of the audience. One third of these adults are parents of the actors. What fraction of the audience are parents of the actors in the play?

Step 1 The phrase " $\frac{1}{3}$ of these adults" tells you that you need to _____.

Step 2 Calculate: $\frac{1}{3} \bullet \frac{2}{5} = \frac{\quad}{\quad}$. So, parents of the actors make up $\frac{\quad}{\quad}$ of the audience at the play.

Step 3 Check your answer for reasonableness. Since $\frac{1}{3}$ is close to $\frac{\quad}{\quad}$ and $\frac{2}{5}$ is close to $\frac{\quad}{\quad}$, their _____ should be about $\frac{\quad}{\quad}$.

Add, subtract, multiply, or divide. Write your answer in simplest form.

23. $\frac{1}{2} + \frac{3}{8}$

24. $\frac{4}{5} - \frac{2}{3}$

25. $1\frac{3}{4} \times \frac{3}{7}$

26. $\frac{5}{8} \div \frac{5}{12}$

27. $2\frac{5}{8} \times 2\frac{1}{7}$

28. $3\frac{1}{4} + 6\frac{1}{3}$

29. $\frac{8}{9} \div 4\frac{2}{3}$

30. $3\frac{5}{6} - 1\frac{1}{2}$

31. $4\frac{1}{8} - 2\frac{3}{4}$

32. $3\frac{1}{5} \div 6\frac{2}{15}$

33. $5\frac{3}{4} + 2\frac{1}{5}$

34. $2\frac{1}{4} \times 1\frac{3}{5}$

35. $7\frac{5}{6} \div 2\frac{7}{12}$

36. $3\frac{1}{10} \times 2\frac{2}{3}$

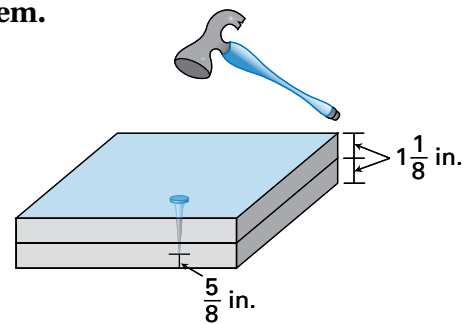
37. $1\frac{2}{3} + 2\frac{5}{6}$

38. $7\frac{2}{7} - 1\frac{3}{5}$

Identify the calculations you must make to solve the problem.
Order the steps you will follow. Then solve the problem.

39. A carpenter hammers a nail into a pair of boards.

Each board is $1\frac{1}{8}$ inches thick. The nail stops $\frac{5}{8}$ inch from of the bottom of the second board. How long is the nail?



40. A chef makes 5 quarts of soup. Each soup bowl holds $\frac{2}{5}$ quart of soup. The chef has filled 7 bowls with soup. How much soup is left?

LESSON 2-18



California Standards

Gr. 2 NS 4.0: Students understand that fractions and decimals may refer to parts of a set and parts of a whole.

Gr. 4 NS 1.6: Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., $\frac{1}{2} = 0.5$ or 0.50 ; $\frac{7}{4} = 1\frac{3}{4} = 1.75$).

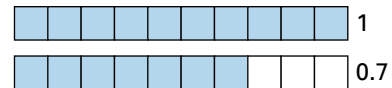
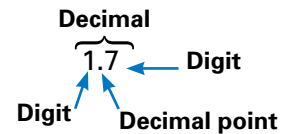
Decimals and Place Value

Words to Remember

Decimal: A number made up of digits and a decimal point

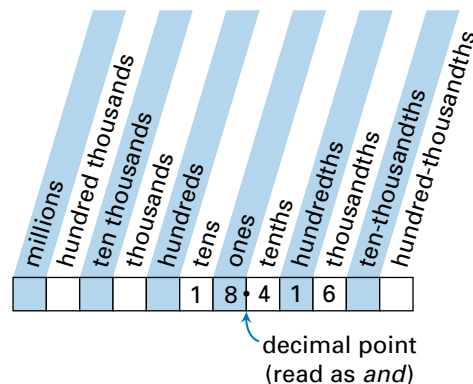
A decimal uses place values to refer to parts of a set or parts of a whole.

You read the decimal 1.7 as “one point seven” or “one and seven tenths.”



Getting Started In Lesson 2-4 you learned how to write and represent fractions. You can also write and represent decimals.

Place Value A decimal is written using place values. Each place value is ten times the place value to its right. Each digit in a decimal has a place value. In the decimal 18.416, the digit 4 is in the tenths' place.



EXAMPLE 1

Identifying Place Value

- Name the digit in the hundredths' place: 2.097
- Name the place value of the digit that is underlined: 41.035

Solution

- The hundredths' place is located two places to the right of the decimal point.
- Three places to the right of the decimal point is the thousandths' place.

2.097
1 2

ANSWER 9

41.035
1 2 3

ANSWER thousandths

TRY THIS

Identify the digit or place value indicated.

- Name the digit in the tenths' place: 43.215
- Name the place value of the digit that is underlined: 1.538

EXAMPLE 2**Reading Decimals****Write the decimal in words.****a.** 5.4**b.** 0.007**Solution**

a. Step 1 Write the whole number part.
The word *and* represents the decimal point.

Words
five and

Numbers
5.4

Step 2 Write the number in the decimal part.

five and four

5.4

Step 3 Write the word for the place value of the rightmost digit.

five and four tenths

5.4
↑
tenths

b. Step 1 Write the word for the decimal part. There is no whole number part.

seven

0.007

Step 2 Write the word for the place value of the rightmost digit.

seven thousandths

0.007
↑
thousandths

Using Zeros

Use zeros to fill in where needed. Since the decimal part in part (a) is seventy-eight *thousandths*, you need to put a zero in the tenths' place so that the eight in seventy-eight is in the thousandths' place.

EXAMPLE 3**Writing Decimals****Write the words as a decimal.****a.** twenty-four and seventy-eight thousandths**b.** six tenths**Solution****a.** twenty-four and seventy-eight thousandths 24.078

The word *and* tells where to put the decimal point.

Thousandths is the place value of the rightmost digit.

b. six tenths

0.6 ← Tenths is the place value of the rightmost digit.

TRY THIS**Write the words as a decimal or the decimal in words.****3.** 241.7**4.** 0.549**5.** three hundred four and twenty-seven thousandths**6.** eighty-two hundredths

Summarize

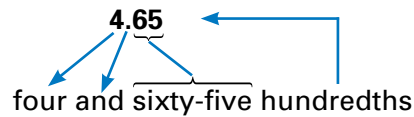
Identifying Place Value

Each place value is ten times the place value to its right.

In 0.34, the 3 is in the tenths' place.

Reading and Writing Decimals

The word *and* represents the decimal point. Numbers before *and* are whole numbers. Numbers after *and* are less than 1. The rightmost digit in a decimal tells you which place value word belongs at the end of the spoken or written form of the decimal.



Practice

Name the digit in the given place value.

1. 48.104; hundredths
2. 7.489; tenths
3. 0.012849; ten thousandths
4. 12.75684; thousandths

Name the place value of the digit that is underlined.

5. 9.4758
6. 52.83497
7. 234.07
8. 200.786
9. 799.5963
10. 24.1538
11. 86.579
12. 64.0183

Match the decimal with its description in words.

13. 271.6 _____
14. 27.16 _____
15. 2.716 _____
- A. twenty-seven and sixteen hundredths
- B. two hundred seventy-one and six tenths
- C. two and seven hundred sixteen thousandths

Write the words as a decimal.

16. eight ten-thousandths
17. two hundred and three tenths
18. twenty-nine thousandths
19. forty-one and five hundredths
20. thirty-five hundredths
21. six and seven thousandths

Write the decimal in words.

22. 3.45
23. 71.404
24. 0.0009
25. 0.16
26. 214.01
27. 392.3
28. 19.002
29. 6.3477

Solve the problem. Explain your reasoning.

- 30.** In the jumping event of a waterskiing tournament, Wesley's air time on his longest jump is **3.48** seconds. Write this decimal in words.

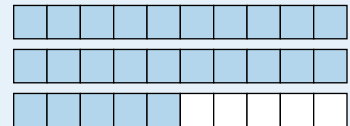
- 31.** Your coach tells you that you ran two and five hundred sixty-two thousandths miles. Write this distance as a decimal.

- 32.** A metric scale in your science class measures the mass of an object as **45.235** grams. Name the digit in the hundredths' place.

DID YOU GET IT?

- 33. Fill in the missing words.** The tenths' place is located _____ place(s) to the _____ of the _____.

- 34. Use a model.** The model represents two and five tenths. Write this as a decimal.



- 35. Supply the number.** Write a decimal that has the digit 5 in the thousandths' place. Then write the decimal in words.

LESSON 2-19



California Standards

Gr. 4 NS 2.2: Round two-place decimals to one decimal or to the nearest whole number and judge the reasonableness of the rounded answer.

Gr. 3/4/5 MR 2.5, Gr. 6 MR 2.6, and Gr. 7 MR 2.7: Indicate the relative advantages of exact and approximate solutions to problems and **give answers to a specified degree of accuracy.**

Round Decimals

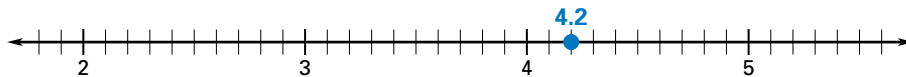
Words to Remember

Rounding: To approximate a number to a given place value

3.2 rounded to the nearest one is 3.

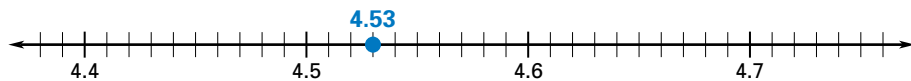
Getting Started You learned how to round whole numbers. You can also round decimals.

Using a Number Line You can see how to round decimals on a number line. Notice that there are 10 equal spaces between each whole number.



EXAMPLE 1 Rounding Decimals Using a Number Line

- a. Use a number line to round 4.53 to the nearest *tenth*.



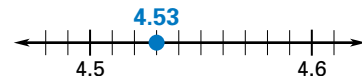
- b. Use a number line to round 0.3497 to the nearest *thousandth*.



Solution

- a. The decimal 4.53 is closer to 4.5 than 4.6, so it rounds down.

ANSWER 4.5



- b. The decimal 0.3497 is closer to 0.350 than 0.349, so it rounds up.

ANSWER 0.35

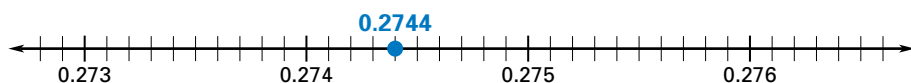


Using Zeros

If the rightmost digit to the right of the decimal point is a zero, you can drop it. So, in part (b), 0.350 is the same as 0.35.

TRY THIS Use a number line.

1. Round 0.2744 to the nearest *thousandth*.



Rounding a Decimal To round a decimal, look at the next digit to the right.

- If the digit is 4 or less, round down.
- If the digit is 5 or greater, round up.

SAMPLE: Round 0.48372 to the place value of the digit that is underlined.

0.48372

The digit 3 is to the right of the 8.

Three is 4 or less, so round down.

0.483 rounds down to 0.48.

EXAMPLE 2

Rounding Decimals

Round the decimal to the place value of the digit that is underlined.

a. 57.651

b. 0.745

Solution

a. The digit to the right of 6 is 5, so 57.651 rounds up to 57.7.

b. The digit to the right of 0 is 7, so 0.745 rounds up to 1.

Rounding When 9 Is in the Given Place Value If the digit in the given place value is 9 and the number to its right is 5 or greater, first change the 9 to a 0. Then increase the digit to the left of 9 by 1.

SAMPLE: Round 2.796 to the nearest hundredth.

2.796

9 is in the hundredths' place, and the digit to its right is a 6.

6 is greater than 5.

2.796 rounds up to 2.80. Change 9 to 0 and increase 7 to 8.

EXAMPLE 3

Rounding to a Given Place Value

Round the decimal to the given place value.

a. 1.3023; hundredths

b. 4.964; tenths

Solution

a. The 0 is in the hundredths' place. The digit to its right is a 2, so 1.3023 rounds down to 1.30, or 1.3.

b. The 9 is in the tenths' place. The digit to its right is a 6, so 4.964 rounds up to 5.0, or 5.

TRY THIS

Round the decimal to the place value of the digit that is underlined.

2. 22.712

3. 16.485

4. 0.315

Summarize

Rounding Decimals

To round to a given place value, look at the next digit to the right. If the digit is 4 or less, round down. If the digit is 5 or greater, round up.

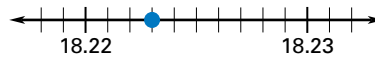


When rounding to the nearest *tenth*, 2.77 rounds up to 2.8.

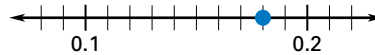
Practice

Use a number line to round the decimal to the given place value.

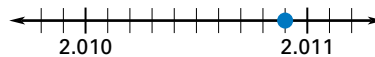
1. 18.223; hundredths _____



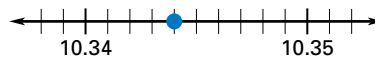
2. 0.18; tenths _____



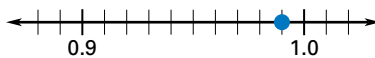
3. 2.0109; thousandths _____



4. 10.344; hundredths _____



5. 0.99; tenths _____



Match the rounded decimal with its original decimal.

6. 0.716 _____

7. 0.72 _____

8. 0.7 _____

- A. 0.7162; tenths

- B. 0.7162; hundredths

- C. 0.7162; thousandths

Round the decimal to the place value of the digit that is underlined.

9. 3.4159

10. 1.9988

11. 0.71274

12. 45.39

13. 24.876

14. 0.4821

15. 30.62

16. 2.797

17. 5.6018

18. 9.551

19. 0.2449

20. 115.02

Round the decimal to the given place value.

21. 5.678; tenths

22. 0.497; hundredths

23. 7.32; ones

24. 0.625; hundredths

25. 1.784; tenths

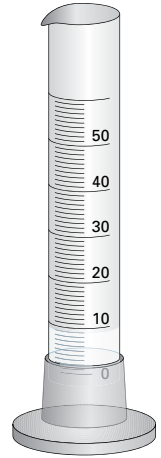
26. 0.0078; thousandths

Solve the problem. Explain your reasoning.

- 27.** You need to measure **8.62** milliliters of a substance for a science experiment. Your lab equipment measures only to the nearest *tenth* of a milliliter. Round **8.62** to the nearest *tenth*.

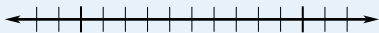
- 28.** You divide **7** by **8** on a calculator and get **0.875**. Round **0.875** to the nearest *hundredth*.

- 29.** A mail clerk tells you that the package you want to mail weighs **8.25** pounds. Round **8.25** pounds to the nearest *pound*.

**DID YOU GET IT?**

- 30. Fill in the missing word.** To round a decimal to a given place value, look at the next digit to the _____.

- 31. Use a number line.** Plot a decimal on the number line. Then give the rounded value when the decimal is rounded to the nearest *tenth*.



Decimal: _____

Rounded to nearest tenth: _____

- 32. Supply the numbers.** Write one decimal that rounds up to **3.18** and one decimal that rounds down to **3.18**.

LESSON
2-20

California Standards

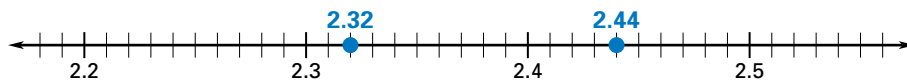
Gr. 4 NS 1.2: Order and compare whole numbers and decimals to two decimal places.

Gr. K/1/2 MR 1.0: Students make decisions about how to set up a problem.

Compare and Order Decimals

Getting Started In Lesson 2-7 you learned how to compare and order fractions. You can also compare and order decimals.

Using a Number Line You can see how to order decimals on a number line. The numbers on a number line increase from left to right, so $2.44 > 2.32$.

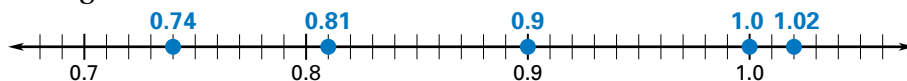

EXAMPLE 1
Ordering Decimals Using a Number Line

Use a number line to order the numbers from least to greatest.

0.81, 1, 0.9, 1.02, 0.74

Solution

Graph each number on a number line. Then list them in order from left to right.

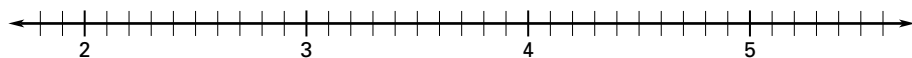


ANSWER 0.74, 0.81, 0.9, 1, 1.02

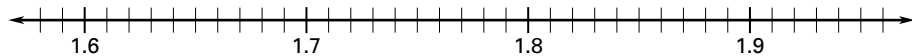
TRY THIS

Use a number line to order the numbers from least to greatest.

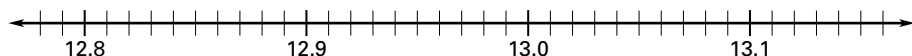
1. 3.3, 4.2, 2, 3, 4.9 _____



2. 1.79, 1.7, 1.66, 1.92, 1.8 _____



3. 13.03, 13.06, 12.93, 12.87, 13 _____



EXAMPLE 2**Comparing Decimals with the Same Number of Decimal Places****Compare the decimals.**

a. 4.61 and 4.62

b. 7.91 and 7.83

Solution

Write the decimals in a column, lining up the decimal places. Compare the place values from left to right. When you come to a place value with different numbers, compare the numbers.

a.

4.61
4.62

same — $\uparrow \uparrow$ — different
 same
 1 < 2

ANSWER 4.61 < 4.62

b.

7.91
7.83

same — $\uparrow \uparrow$ — different
 9 > 8

ANSWER 7.91 > 7.83

TRY THIS Compare the decimals.

4. 14.42 and 14.47

5. 0.31 and 0.29

EXAMPLE 3**Comparing Decimals with a Different Number of Decimal Places****Compare the decimals: 5.02 and 5****Solution**

Step 1 Write the decimals in a column, lining up the decimal places.

Step 2 Add zeros so that each decimal has the same number of decimal places.

5.02
5.00

same — $\uparrow \uparrow$ — different
 same

Add zeros after 5 to match 5.02.

Step 3 Compare the place values from left to right.

ANSWER 2 > 0, so 5.02 > 5

TRY THIS Compare the decimals.

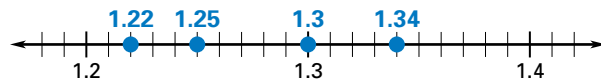
6. 0.16 and 0.092

7. 3 and 2.99

Summarize

Ordering Decimals

Graph the numbers on a number line. List the numbers in order from left to right (least to greatest).



The numbers in order from least to greatest are **1.22, 1.25, 1.3, 1.34**.

Comparing Decimals

Write the decimals in a column. If the decimals have a different number of decimal places, add zeros to make the number of decimal places the same. Compare the place values from left to right.

3.48 and 3.47

3.48

3.47

$3.48 > 3.47$

0.5 and 0.55

0.50

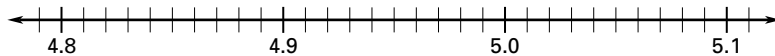
0.55

$0.5 < 0.55$

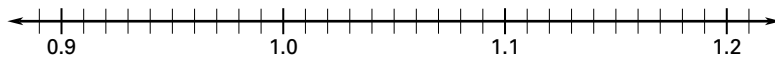
Practice

Use a number line to order the numbers from least to greatest.

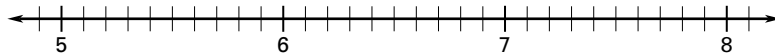
1. 4.84, 4.94, 4.82, 5 _____



2. 1.08, 0.98, 1, 0.96 _____

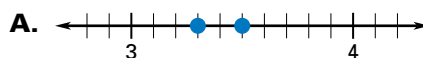


3. 7, 5, 7.7, 6.3 _____



Match the statement with the number line that represents it.

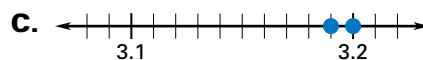
4. $3.5 > 3.3$ _____



5. $3.19 < 3.2$ _____



6. $3.15 > 3.1$ _____



Complete the statement with $<$, $>$, or $=$.

7. $0.7 \bigcirc 0.6$

8. $9.12 \bigcirc 9.14$

9. $16.92 \bigcirc 16.91$

10. $0.81 \bigcirc 0.9$

11. $1.7 \bigcirc 1.74$

12. $21 \bigcirc 21.4$

13. $8.1 \bigcirc 8.01$

14. $6.29 \bigcirc 6$

15. $7 \bigcirc 6.54$

16. $0.8 \bigcirc 0.80$

17. $42.15 \bigcirc 42.18$

18. $0.45 \bigcirc 0.06$

19. $1.35 \bigcirc 1.31$

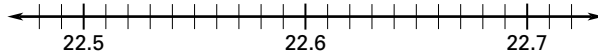
20. $12.6 \bigcirc 12.07$

21. $0.70 \bigcirc 0.74$

Solve the problem. Explain your reasoning.

- 22.** In a long jump competition, Mario jumped **4.45** feet and Cole jumped **4.49** feet. Who jumped farther?

- 23.** Order the dogs from lightest to heaviest.



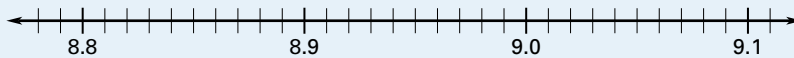
Dog	Weight (pounds)
Bonnie	22.6
Clyde	22.58
Mona	22.71
Lisa	22.68

- 24.** You type a paragraph in **3.68** minutes. Your friend types the same paragraph in **3.6** minutes. Who types faster, you or your friend?

DID YOU GET IT?

- 25. Fill in the missing words.** To compare two decimals, write the decimals in a(n) _____. Then compare the _____ from _____ to _____.

- 26. Use a number line.** Plot four numbers on the number line. Then list the numbers in order from least to greatest.



- 27. Supply the number.** Write a decimal with two decimal places that is less than **0.1**.

LESSON 2-21



California Standards

Gr. 3 NS 3.4: Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $\frac{1}{2}$ of a dollar, 75 cents is $\frac{3}{4}$ of a dollar).

Gr. 7 NS 1.5: Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

Also included: Gr. 4 NS 1.6

Write Decimals as Fractions

Getting Started In Lesson 2-8 you learned how to write mixed numbers as improper fractions. You can also write decimals as fractions and mixed numbers.

Using Place Value Use the place value of the rightmost digit in a decimal to help you write the decimal as a fraction. The place value of the rightmost digit tells the *denominator* of the fraction.

$$\begin{aligned} 0.\underline{1} &\rightarrow \text{one } \textit{tenth} \rightarrow \frac{1}{10} \\ 0.0\underline{1} &\rightarrow \text{one } \textit{hundredth} \rightarrow \frac{1}{100} \\ 0.00\underline{1} &\rightarrow \text{one } \textit{thousandth} \rightarrow \frac{1}{1000} \end{aligned}$$

Using Symbols

You can use the cent sign or the dollar sign to represent one penny. 1¢ or \$.01

EXAMPLE 1

Writing Decimals as Fractions

Write the decimal as a fraction in simplest form.

a. 0.7

b. 0.42

Solution

a. Write 0.7 as a fraction.

$$0.\underline{7} \rightarrow 7 \text{ tenths} \rightarrow \frac{7}{10}$$

$$\frac{7}{10} \text{ is in simplest form.}$$

$$\text{ANSWER } \frac{7}{10}$$

b. Write 0.42 as a fraction.

$$0.\underline{42} \rightarrow 42 \text{ hundredths} \rightarrow \frac{42}{100}$$

$$\frac{42 \div 2}{100 \div 2} = \frac{21}{50} \quad \leftarrow \text{Use the GCF to simplify.}$$

$$\text{ANSWER } \frac{21}{50}$$

Remember

To determine whether a fraction is in simplest form, look at the numerator and denominator. If they share a common factor, then the fraction is *not* in simplest form.

TRY THIS

Write the decimal as a fraction in simplest form.

1. $0.3 = \frac{\square}{\square}$

2. $0.27 = \frac{\square}{\square}$

3. $0.115 = \frac{\square}{\square}$

4. $0.75 = \frac{\square}{\square}$

Recognizing Common Decimals and Their Fraction

Equivalents You can use what you know about money to help you recognize common decimals and their fraction equivalents.

$$\$0.25 \text{ is } \frac{1}{4} \text{ of a dollar} \rightarrow 0.25 = \frac{1}{4}$$

$$\$0.50 \text{ is } \frac{1}{2} \text{ of a dollar} \rightarrow 0.5 = \frac{1}{2}$$

$$\$0.75 \text{ is } \frac{3}{4} \text{ of a dollar} \rightarrow 0.75 = \frac{3}{4}$$

EXAMPLE 2**Writing Decimals as Mixed Numbers**

Write the decimal as a mixed number in simplest form.

a. 1.6

b. 2.25

Solution

a. $1\frac{6}{10}$

Write 1.6, or one and six tenths, as a mixed number.

$$1\frac{6 \div 2}{10 \div 2} = 1\frac{3}{5}$$

Use the GCF to simplify the fraction part.

ANSWER $1\frac{3}{5}$

b. $2.25 = 2 + 0.25$

You can break 2.25 into 2 and the decimal 0.25.

$$0.25 = \frac{1}{4}$$

0.25 is equal to one fourth ($0.25 = \frac{1}{4}$).

$$2 + 0.25 = 2 + \frac{1}{4}$$

Two and twenty-five hundredths is the same as two and one fourth.

ANSWER $2\frac{1}{4}$

Write two and one fourth as a mixed number.

EXAMPLE 3**Writing Decimals with Zeros as Fractions or Mixed Numbers**

Write the decimal as a fraction or mixed number in simplest form.

a. 0.006

b. 5.09

Solution

a. $\frac{6}{1000}$

Write 0.006, or six thousandths, as a fraction.

$$\frac{6 \div 2}{1000 \div 2} = \frac{3}{500}$$

Use the GCF to simplify.

ANSWER $\frac{3}{500}$

b. $5\frac{9}{100}$

Write 5.09, or five and nine hundredths, as a mixed number. The mixed number is in simplest form.

TRY THIS

Write the decimal as a fraction or mixed number in simplest form.

5. $7.5 = \frac{\square}{\square}$

6. $4.45 = \frac{\square}{\square}$

7. $0.02 = \frac{\square}{\square}$

Summarize**Writing Decimals as Fractions or Mixed Numbers**

- | | | |
|---|------|---|
| (1) Use place value to write the decimal as a fraction or mixed number. | 0.59 | fifty-nine <i>hundredths</i>
$= \frac{59}{100}$ |
| (2) Learn to recognize common decimals and their fraction equivalents. | 1.75 | $0.75 = \frac{3}{4}$, so 1.75 is one and three fourths
$= 1\frac{3}{4}$ |
| (3) Simplify if necessary. | 2.05 | two and five <i>hundredths</i>
$= 2\frac{5}{100} = 2\frac{1}{20}$ |

Practice

Match the decimal with the equivalent fraction in simplest form.

- | | | | |
|------------------|------------------|------------------|------------------|
| 1. 0.5 _____ | 2. 0.25 _____ | 3. 0.75 _____ | 4. 0.125 _____ |
| A. $\frac{3}{4}$ | B. $\frac{1}{8}$ | C. $\frac{1}{4}$ | D. $\frac{1}{2}$ |

Match the decimal with the equivalent mixed number in simplest form.

- | | | | |
|-------------------|---------------------|-------------------|--------------------|
| 5. 3.125 _____ | 6. 3.8 _____ | 7. 3.64 _____ | 8. 3.06 _____ |
| A. $3\frac{4}{5}$ | B. $3\frac{16}{25}$ | C. $3\frac{1}{8}$ | D. $3\frac{3}{50}$ |

Write the decimal as a fraction in simplest form.

- | | | | |
|---------|-----------|-----------|----------|
| 9. 0.12 | 10. 0.4 | 11. 0.625 | 12. 0.35 |
| 13. 0.9 | 14. 0.46 | 15. 0.95 | 16. 0.28 |
| 17. 0.2 | 18. 0.255 | 19. 0.175 | 20. 0.6 |

Write the decimal as a fraction or mixed number in simplest form.

- | | | | |
|----------|-----------|-----------|-----------|
| 21. 9.15 | 22. 1.1 | 23. 3.315 | 24. 4.25 |
| 25. 2.76 | 26. 8.5 | 27. 2.02 | 28. 3.008 |
| 29. 0.09 | 30. 0.004 | 31. 7.65 | 32. 5.042 |

Solve the problem. Explain your reasoning.

- 33.** You walk **0.25** mile and then jog **0.75** mile. Write these decimals as fractions.

- 34.** It takes you **3.04** minutes to download a song from an online music store. Write this decimal as a mixed number.

- 35.** A scrap piece of wood is one and seven tenths of a meter long. Write this number as a decimal and as a mixed number.

DID YOU GET IT?

- 36. Fill in the missing words.** When writing a decimal as a fraction, the _____ tells the _____ of the fraction.

- 37. Find the error.** Your friend says that to write **0.08** as a fraction, you should write eight tenths as $\frac{8}{10}$ and then simplify to $\frac{4}{5}$. Is your friend correct? Explain.

- 38. Supply the number.** Write a decimal so that its rightmost digit is in the hundredths' place. Then write your decimal as a fraction or mixed number.

LESSON 2-22



California Standards

Gr. 4 NS 1.7: Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and **relate a fraction to a simple decimal on a number line.**

Gr. 5 NS 1.2: Interpret percents as a part of a hundred; **find decimal** and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.

Also included: Gr. 3/4/5 MR 2.5, Gr. 6 MR 2.6, and Gr. 7 MR 2.7

Write Fractions as Decimals

Words to Remember

Terminating decimal: A decimal that has a final digit

0.2 and 0.47 are terminating decimals.

Repeating decimal: A decimal that has one or more digits that repeat forever

You can write a repeating decimal with a bar over the digits that repeat.

0.333... and 0.4545... are repeating decimals and can be written as $0.\overline{3}$ and $0.\overline{45}$.

Getting Started In Lesson 2-21 you learned how to write decimals as fractions. You can also write fractions as decimals.

EXAMPLE 1

Writing a Fraction as a Terminating Decimal

Write the fraction or mixed number as a decimal.

a. $\frac{13}{25}$

b. $1\frac{4}{5}$

Solution

$$\begin{array}{r} 0.52 \\ \text{a. } 25 \overline{)13.00} \\ \underline{12.5} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

Divide the numerator by the denominator.

The remainder is 0 because the decimal is *terminating*.

ANSWER 0.52

b. $1\frac{4}{5} = \frac{(1 \times 5) + 4}{5} = \frac{9}{5}$

Write the mixed number as an improper fraction.

$$\begin{array}{r} 1.8 \\ 5 \overline{)9.0} \\ \underline{5} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Divide the numerator by the denominator. The remainder is 0 because the decimal is *terminating*.

ANSWER 1.8

TRY THIS

Write the fraction or mixed number as a decimal.

1. $\frac{7}{20}$

2. $\frac{9}{40}$

3. $5\frac{3}{4}$

4. $1\frac{3}{16}$

EXAMPLE 2**Writing a Fraction as a Repeating Decimal**Write $\frac{6}{11}$ as a decimal.**Solution**

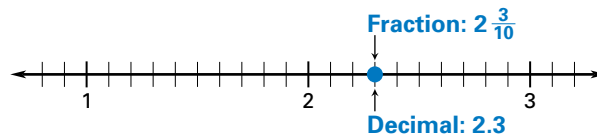
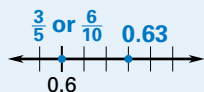
$$\begin{array}{r}
 0.5454\ldots \\
 11 \overline{) 6.0000\ldots} \\
 \underline{55} \\
 50 \\
 \underline{44} \\
 60 \\
 \underline{55} \\
 50 \\
 \underline{44} \\
 6
 \end{array}$$

Divide the numerator by the denominator.

The digits 5 and 4 repeat. Write a bar over the 5 and 4.

Using a Decimal Bar

Remember, you can write repeating decimals with a bar over the digits that repeat.

ANSWER $0.\overline{54}$ **Comparing Fractions and Decimals** You can see how decimals and fractions relate on a number line. To compare a fraction and a decimal, write the fraction as a decimal.**Using a Number Line**You can check your comparison by graphing the fraction and decimal on a number line. Multiply the numerator and denominator of $\frac{3}{5}$ by 2 to get $\frac{6}{10}$.**EXAMPLE 3****Comparing Fractions and Decimals**Compare: $\frac{3}{5}$ and 0.63**Solution**

$$\begin{array}{r}
 0.6 \\
 5 \overline{) 3.0} \\
 \underline{30} \\
 0
 \end{array}$$

Write $\frac{3}{5}$ as a decimal.

$0.6 < 0.63$

Compare the decimals.

ANSWER $\frac{3}{5} < 0.63$ **TRY THIS**

5. Write $\frac{2}{3}$ as a repeating decimal. 6. Compare: $1\frac{3}{8}$ and 1.36

Summarize**Writing Fractions or Mixed Numbers as Decimals**

For a decimal, divide the numerator by the denominator. For a mixed number, write the mixed number as an improper fraction. Then divide the numerator by the denominator. Write repeating decimals with a bar over the digits that repeat.

$$\frac{4}{25}$$

$$\begin{array}{r} 0.16 \\ 25 \overline{)4.00} = 0.16 \\ \underline{25} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

$$\frac{1}{3}$$

$$\begin{array}{r} 0.333... \\ 3 \overline{)1.000...} = 0.\overline{3} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Comparing Fractions and Decimals

Write the fraction as a decimal. Then compare the decimals.

$$\begin{array}{l} \text{Compare } \frac{9}{10} \text{ and } 0.94. \rightarrow \frac{9}{10} = 0.9 \\ 0.9 < 0.94 \\ \frac{9}{10} < 0.94 \end{array}$$

Practice

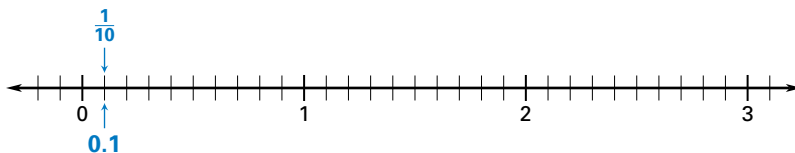
Graph each fraction or mixed number on the number line below. Then give the decimal equivalent.

1. $\frac{9}{10}$ _____

2. $\frac{1}{2}$ _____

3. $1\frac{7}{10}$ _____

4. $2\frac{3}{5}$ _____



Match the fraction or mixed number with an equivalent decimal.

5. $\frac{4}{15}$ _____

6. $\frac{17}{20}$ _____

7. $1\frac{5}{18}$ _____

8. $1\frac{4}{5}$ _____

A. 1.8

B. 0.85

C. $0.2\overline{6}$

D. $1.2\overline{7}$

Write the fraction or mixed number as a decimal.

9. $\frac{2}{5}$

10. $\frac{3}{32}$

11. $1\frac{11}{40}$

12. $2\frac{13}{50}$

13. $\frac{1}{12}$

14. $\frac{7}{24}$

15. $4\frac{2}{3}$

16. $1\frac{4}{11}$

Complete the statement with $<$, $>$, or $=$.

17. $\frac{6}{25}$ ☐ 0.24

18. $\frac{19}{20}$ ☐ 0.97

19. $2\frac{3}{8}$ ☐ 2.37

20. $3\frac{1}{4}$ ☐ 3.28

21. $1\frac{1}{16}$ ☐ 1.05

22. $\frac{5}{80}$ ☐ 0.07

Solve the problem. Explain your reasoning.

- 23.** You are mailing a letter that weighs $\frac{7}{8}$ of a pound. Write $\frac{7}{8}$ as a decimal.

- 24.** You measure the height of a young magnolia tree. Write the height as a decimal.

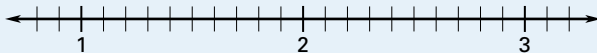


- 25.** You are picking a colored paper to print a dozen flyers. The blue paper is $4\frac{7}{16}$ inches wide, and the yellow paper is 4.35 inches wide. Which paper is wider?

DID YOU GET IT?

- 26. Fill in the missing word.** A decimal that has a final digit is called a _____ decimal.

- 27. Use a number line.** Graph a fraction on the number line. Then write a decimal that is equivalent to the fraction.



Fraction: _____

Decimal: _____

- 28. Solve the riddle.** Three fractions with 12 in the denominator can be written as terminating decimals. Which fractions are they? (Hint: The fraction does not need to be in simplest form.)

Mixed Practice for Lessons 2-18 to 2-22

Vocabulary Review

Match the word with its definition.

Word	Definition
1. round _____	A. describes a decimal that has a final digit
2. terminating _____	B. to write a decimal to a given place value
3. whole _____	C. one unit

Fill in the missing word(s).

4. The decimal 0.5333... is a _____ because the 3 occurs over and over forever.
5. The _____ of a decimal tells the denominator of its fraction equivalent.

Write the words as a decimal or the decimal in words.

6. fourteen and sixty-five thousandths 7. 105.61

Name the digit in the given place value. Then use the number line to round the decimal to the given place value.

8. 2.8375; hundredths

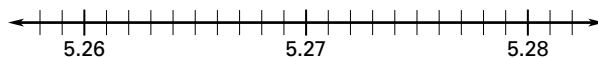


9. 1.96; tenths

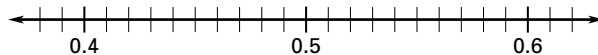


Use the number line to order the numbers from least to greatest.

10. 5.268, 5.26, 5.271, 5.27



11. 0.6, 0.45, 0.52, 0.5



Complete the statement with $<$, $>$, or $=$.

12. $25.4 \bigcirc 26.96$

13. $30.1 \bigcirc 30.01$

14. $7.2 \bigcirc 7\frac{1}{3}$

15. $4\frac{1}{4} \bigcirc 4.15$

16. $\frac{9}{10} \bigcirc 0.9$

17. $1.31 \bigcirc 1\frac{2}{7}$

- 18.** Fill in the missing information to solve the problem.

In gym class today, Meredith jumped a distance of **5.24** feet. Lydian jumped a distance of $5\frac{1}{4}$ feet. Who jumped the shorter distance?

Step 1 The word “shorter” tells you that you need to compare **5.24** and $5\frac{1}{4}$ to see which one is _____.

Step 2 Compare: **5.24** ● $5\frac{1}{4}$

Step 3 _____ jumped the shorter distance.

Write the decimal as a fraction or mixed number in simplest form.

19. 0.75

20. 0.02

21. 0.58

22. 3.6

23. 4.15

24. 2.84

25. 9.9

26. 0.005

Write the fraction or mixed number as a decimal.

27. $\frac{73}{100}$

28. $\frac{3}{8}$

29. $\frac{2}{25}$

30. $\frac{17}{20}$

31. $3\frac{1}{8}$

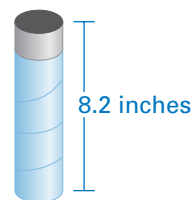
32. $1\frac{2}{5}$

33. $5\frac{61}{100}$

34. $1\frac{3}{4}$

Solve the problem. Explain your reasoning.

- 35.** Jaime is mailing a rolled poster with a height of $8\frac{2}{5}$ inches. Will it fit into this mailing tube?



- 36.** At a grocery store, you order **4.25** pounds of ground turkey. The butcher rounds **4.25** to the nearest tenth and gives you that amount of ground turkey. Do you get more or less ground turkey than you ordered? Explain.

LESSON 2-23



California Standards

Gr. 4 NS 2.0: Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals.

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Also included: Gr. 5 NS 2.0

Add and Subtract Decimals

Words to Remember

Regroup: To write a group of numbers as another number

214 can be regrouped as $\overset{1}{2} \overset{11}{\cancel{1}} 4$, or 1 hundred, 11 tens, and 4 ones.

Getting Started In Lessons 2-11 and 2-12, you learned how to add and subtract fractions. You can also add and subtract decimals.

Adding Decimals To add decimals, line up the decimal points. Then add the same way you would with whole numbers. Bring the decimal point down in your answer. You can add two or more numbers following the same steps.

$$\begin{array}{r} 5.2 \\ + 5.3 \\ \hline 10.5 \end{array}$$

Remember

\$ is the sign for dollar.

¢ is the sign for cents.

\$ 4.29

+ 7.18

\$11.97

You say this:

"Eleven dollars and ninety seven cents".

EXAMPLE 1

Adding Decimals with the Same Number of Decimal Places

Add: $4.29 + 7.18$

Solution

$$\begin{array}{r} 1 \\ 4.29 \\ + 7.18 \\ \hline 11.97 \end{array}$$

Write the decimals in a column, lining up the decimal points.

Add, regrouping as necessary. Bring the decimal point down.

ANSWER 11.97

EXAMPLE 2

Adding Decimals with a Different Number of Decimal Places

Add: $5.061 + 2.3$

Solution

$$\begin{array}{r} 5.061 \\ + 2.3 \\ \hline 5.061 \\ + 2.300 \quad \leftarrow \text{Add zeros.} \\ \hline 7.361 \end{array}$$

Write the decimals in a column, lining up the decimal points or the digits in the ones' place.

Add zeros so that both numbers have the same number of digits after the decimal.

Add, bringing the decimal point down.

ANSWER 7.361

TRY THIS Add.

1. $5.956 + 1.714$

2. $3.292 + 14.5$

Subtracting Decimals To subtract decimals, line up the decimal points. Then subtract the same way you would with whole numbers. Bring the decimal point down in your answer.

$$\begin{array}{r} 3.8 \\ - 1.5 \\ \hline 2.3 \end{array}$$

EXAMPLE 3**Subtracting Decimals with the Same Number of Decimal Places**

Subtract: $3.16 - 1.79$

Solution

$$\begin{array}{r} \overset{2}{\cancel{3}}.\overset{11}{\cancel{1}6} \\ - 1.76 \\ \hline 1.40 \end{array}$$

Don't Forget

Drop the final digit if it is a zero. So, 1.40 becomes 1.4.

Write the decimals in a column, lining up the decimal points.

Subtract, regrouping if necessary. Bring the decimal point down. The final digit is a zero, so you can drop it.

ANSWER 1.4**EXAMPLE 4****Subtracting Decimals with a Different Number of Decimal Places**

Subtract: $9 - 5.375$

Solution

$$\begin{array}{r} 9 \\ - 5.375 \\ \hline 3.625 \end{array}$$

Write the numbers in a column. If one of the numbers is a whole number, line up the digits in the ones' place.

Add a decimal point to the whole number and zeros to the right of the decimal point. Both numbers should have the same number of digits after the decimal point.

Subtract, bringing the decimal point down.

ANSWER 3.625**TRY THIS** Subtract.

3. $4.116 - 2.004$

4. $27.1 - 18.2$

5. $16 - 0.578$

6. $6.44 - 3.9$

Summarize**Adding Decimals**

Write the decimals in a column, lining up the decimal points.
Add zeros to the right of the decimal point if necessary. Add,
bringing the decimal point down.

$$\begin{array}{r} 2.25 + 4.2 \\ 2.25 \\ + 4.20 \\ \hline 6.45 \end{array}$$

Subtracting Decimals

Write the decimals in a column, lining up the decimal points.
Add zeros to the right of the decimal point if necessary.
Subtract, bringing the decimal point down.

$$\begin{array}{r} 7.8 - 1.82 \\ 7.80 \\ - 1.82 \\ \hline 5.98 \end{array}$$

Practice

Match the addition statement with the sum.

- | | |
|-------------------------------|----------|
| 1. $7.007 + 0.7 + 1.07$ _____ | A. 7.067 |
| 2. $7.107 + 0.07 + 0.6$ _____ | B. 7.777 |
| 3. $7.07 + 0.6 + 0.1$ _____ | C. 8.777 |
| 4. $7 + 0.06 + 0.007$ _____ | D. 7.77 |

Add.

- | | | |
|--------------------|-------------------|--------------------|
| 5. $9.171 + 4.784$ | 6. $4.78 + 0.25$ | 7. $1.24 + 8.87$ |
| 8. $14.7 + 22.4$ | 9. $10.04 + 5.8$ | 10. $0.009 + 2.48$ |
| 11. $6.781 + 8$ | 12. $1.785 + 4.2$ | 13. $5.2 + 0.08$ |

Match the subtraction statement with the difference.

- | | |
|---------------------------|----------|
| 14. $9.99 - 4.78$ _____ | E. 5.19 |
| 15. $9.909 - 4.181$ _____ | F. 5.21 |
| 16. $9.9 - 4.71$ _____ | G. 5.728 |

Subtract.

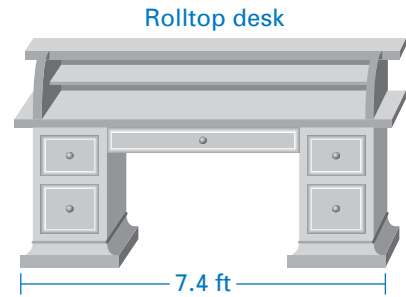
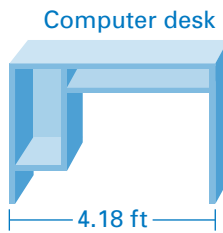
- | | | |
|---------------------|--------------------|-------------------|
| 17. $8.359 - 3.128$ | 18. $5.67 - 3.22$ | 19. $2.5 - 0.2$ |
| 20. $24.3 - 10.3$ | 21. $58.4 - 2$ | 22. $3.02 - 0.78$ |
| 23. $11 - 4.77$ | 24. $6.235 - 4.04$ | 25. $4.5 - 2.12$ |

Name _____

Date _____

Solve the problem. Explain your reasoning.

- 26.** What is the difference in the lengths of the two desks?



- 27.** You have \$10.74. Your friend gives you \$4.75. How much do you have now?

- 28.** Your driveway is 25.375 feet long. Your neighbor's driveway is 78.25 feet long. How much longer is your neighbor's driveway?

DID YOU GET IT?

- 29. Fill in the missing words.** To add or subtract decimals, you must write them in a column, lining up the _____.

- 30. Find the error.** Halley uses the method shown to add $12.14 + 1.03$. Is she correct? Why or why not?

12.14
+ 1.030
2.244

- 31. Write a problem.** Write and solve a subtraction problem with decimals that have a different number of decimal places.

LESSON 2-24



California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Multiply Decimals by Whole Numbers

Words to Remember

Power: An expression, such as 3^2 , that represents a product with a repeated factor

Power of 10: In a power of ten, the repeated factor is 10. For example, $10^1 = 10 \times 1 = 10$ and $10^3 = 10 \times 10 \times 10 = 1000$. A number with 1 in the first place value followed by any number of zeros is a power of 10.

Getting Started You learned how to multiply whole numbers. You can also multiply decimals by whole numbers.

Multiplying by Powers of Ten To multiply a decimal by a power of ten, first count the zeros in the power of ten. Then move the decimal point one place to the right for each zero.

$$4.756 \times 100 = 475.6$$

There are 2 zeros. $\uparrow \quad \uparrow \quad \uparrow$
1 2

EXAMPLE 1

Multiplying Decimals by Powers of Ten

Multiply.

a. 0.1467×10

b. 4.3×1000

Solution

a. 0.1467×10

Count the number of zeros in the power of ten.

There is 1 zero. \uparrow

01.467

Move the decimal point one place to the right for each zero in the power of ten.

\downarrow
1

1.467

Drop the leftmost zero because the number is greater than 1.

ANSWER 1.467

b. 4.3×1000

Count the number of zeros in the power of ten.

There are 3 zeros. \uparrow

4.300

Add zeros to the right of the decimal.

4300

Move the decimal point one place to the right for each zero in the power of ten.

$\downarrow \downarrow \downarrow$
1 2 3

ANSWER 4300

TRY THIS Multiply.

1. 1.2×100

2. 0.007×10

Finding the Product of a Decimal and a Whole Number A *product* is the result of multiplying two numbers. A *factor* is one of the numbers being multiplied. The *product* of a decimal and a whole number has the same number of decimal places as the *decimal factor*.

	0.002	← Decimal factor
3 decimal places	$\times 3$	← Whole number factor
	0.006	← Product

EXAMPLE 2**Multiplying a Decimal by a Whole Number**Multiply: 0.09×3 **Solution****Remember**

Drop any final zeros after you place the decimal point.

Example: 0.004×5 is 0.020, or 0.02.

Step 1 Write the decimals in a column. You do not need to line up the decimal points or the ones' digits.

$$\begin{array}{r} 0.09 \\ \times 3 \\ \hline \end{array}$$

Step 2 Multiply as you would whole numbers.

Step 3 Decide how many decimal places are in the product. There are two decimal places in **0.09**. So, the product also needs two decimal places.

$$\begin{array}{r} 0.09 \\ \times 3 \\ \hline 27 \end{array}$$

Step 4 Count to the left from the rightmost digit of the product. Because **0.09** has two decimal places, count 2 digits to the left. Place the decimal point.

$$\begin{array}{r} .27 \\ \times 3 \\ \hline 81 \end{array}$$

Step 5 Add a zero to the left of the decimal if necessary.

$$0.27$$

ANSWER 0.27**TRY THIS** Multiply.

3. 1.25×3

$$\begin{array}{r} 1.25 \\ \times 3 \\ \hline \end{array}$$

____ decimal places

4. 0.008×5


$$\begin{array}{r} 0.008 \\ \times 5 \\ \hline \end{array}$$

____ decimal places

5. A quarter is worth 25¢, and you have two. How much money do you have? Write your answer two ways. \$. __ __ ¢

Summarize**Multiplying a Decimal by a Power of Ten**

Count the number of zeros in the power of ten. Add zeros to the right of the decimal if necessary. Move the decimal point one place to the right for each zero in the power of ten. Drop the leftmost zero if the product is greater than or equal to 1.

$$0.75 \times 1000 = 750$$


Multiplying a Decimal by a Whole Number

Write the decimals in a column. Multiply. Place the decimal point so that the product has the same number of decimal places as the decimal factor. Add a zero to the left of the decimal if necessary. Drop any rightmost zeros if necessary.

$$\begin{array}{r} 1.5 \times 2 \\ \times 2 \\ \hline 30 \\ 3 \end{array}$$

Practice

Match the multiplication statement with the product.

- | | |
|-------------------------------|-----------|
| 1. 1000×14.487 _____ | A. 0.06 |
| 2. 14.487×100 _____ | B. 5.1 |
| 3. 8.01×4 _____ | C. 32.04 |
| 4. 5×0.012 _____ | D. 1448.7 |
| 5. 3×1.7 _____ | E. 14,487 |

Multiply by a power of 10.

- | | | |
|------------------------|--------------------------|------------------------|
| 6. 4.729×10 | 7. 7.79×100 | 8. 12.5×10 |
| 9. 29.93×1000 | 10. 95.2×100 | 11. 144.03×10 |
| 12. 56.1×1000 | 13. 9.7913×1000 | 14. 8.4×100 |

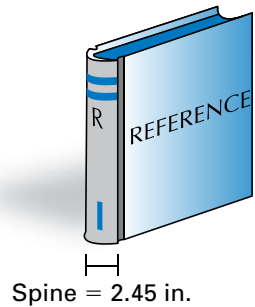
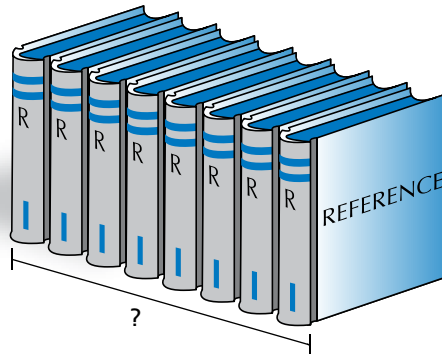
Multiply.

- | | | |
|----------------------|-----------------------|-----------------------|
| 15. 0.004×3 | 16. 0.08×14 | 17. 9×0.9 |
| 18. 0.016×5 | 19. 7×0.28 | 20. 4.5×8 |
| 21. 2×1.014 | 22. 1.343×6 | 23. 2.002×25 |
| 24. 5×2.24 | 25. 2.603×4 | 26. 32×0.005 |
| 27. 4.001×3 | 28. 12×2.306 | 29. 9×4.52 |

Solve the problem. Explain your reasoning.

- 30.** One eraser costs **\$.03**. How much does a pack of **100** erasers cost?

- 31.** The length across the spine of a reference book is **2.45** inches. If you place **8** of these books on a shelf side by side, what will the total length be?



- 32.** One box of cereal weighs **0.8125** pound. If you put **30** boxes of this cereal in a crate, how much weight will the crate contain?

DID YOU GET IT?

- 33. Fill in the missing words.** When multiplying a decimal by a whole number power of ten, move the decimal point one place to the _____ for each _____ in the power of ten.

- 34. Find the error.** Vladimir uses the method shown at the right to multiply 15.4×9 . Is he correct? Why or why not?

15.4
$\times 9$
1.386

ACTIVITY 2-25



California Standards

Gr. K/1/2 MR 1.2: Use tools (Gr. K/1/2) and strategies (Gr. K), such as manipulatives or sketches, to model problems.

Gr. K MR 2.1: Explain the reasoning used with concrete objects and/or pictorial representations.

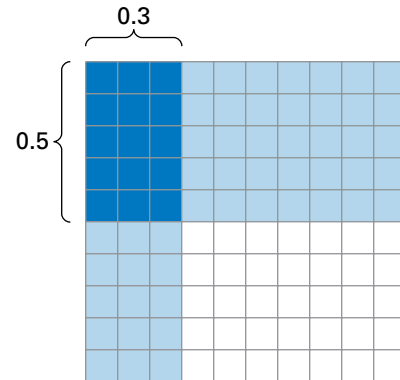
Also included: Gr. 5 NS 2.0

Multiplying Decimals Using Models

Goal: Use area models to understand how to multiply decimals.

Getting Started You can use an area model to model multiplying decimals. Shade columns in the area model to represent one decimal. Shade rows in the area model to represent the other decimal. The squares that have the double shading represent the product of the two decimals.

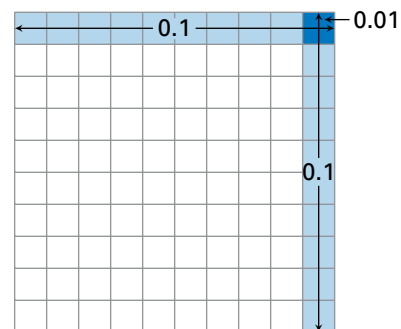
$$0.5 \times 0.3 = 0.15$$



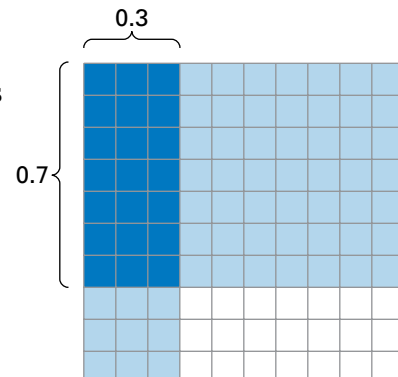
EXAMPLE

Use the following steps to model the product: 0.7×0.3

Step 1 Make a grid of 100 squares. The whole square represents 1. Each row and column of squares represents 0.1. Each small square represents 0.01.



Step 2 Shade 7 rows of squares to represent 0.7. Then shade 3 columns to represent 0.3. There are 21 squares that have double shading.



Step 3 Write the result.

$$0.7 \times 0.3 = 0.21$$

MAKE IT A GAME!

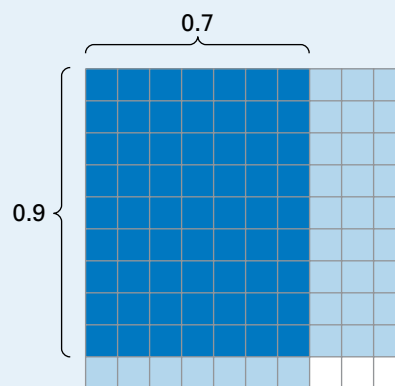
- Form groups of three students. Give each student two blank grids.
- In each group, have two students think of two different decimals between **0** and **0.9**.
- Have the third student model the product of the two decimals.
- Change roles within the group. Each student should get two chances to model a product.
- Tell the groups that each of their six collective products must be unique. The first group to accomplish this correctly wins.

First student says “**0.9**.”

Second student says “**0.7**.”

Third student models the product.

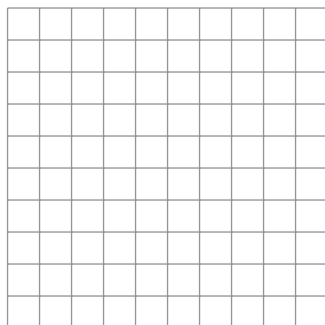
$$0.9 \times 0.7 = 0.63$$



Practice

In Exercises 1 and 2, use models to multiply the decimals.

1. 0.4×0.4



2. 0.9×0.5



In Exercises 3–5, draw models to multiply the decimals. Then write the product.

3. 0.2×0.6

4. 0.4×0.3

5. 0.5×0.7

- 6. Make a Conjecture** Based on your answers to Exercises 1–5, make a conjecture about how to multiply two decimals without using models.

LESSON 2-26



California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Multiply Decimals

Getting Started In Lesson 2-24 you learned how to multiply decimals by whole numbers. You can also multiply two decimals.

Multiplying Decimals To multiply decimals, write them in a column. Then multiply the same way you would whole numbers. Place the decimal point so that the product has the same number of decimal places as the total number of decimal places in the factors.

$$\begin{array}{r} 2.06 \\ \times 3.4 \\ \hline 824 \\ + 6180 \\ \hline 7.004 \end{array}$$

There are a total of $2 + 1 = 3$ decimal places in the factors.

There are 3 decimal places in the product.

EXAMPLE 1

Placing a Decimal Point in a Product

Place the decimal point in the product.

a. $9.523 \times 5.43 = 5170989$

b. $0.35 \times 0.2 = 70$

Solution

a. **Step 1** Count the total number of decimal places in the factors. $9.523 \times 5.43 = 5170989$
5 decimal places

Step 2 Count the same number of decimal places in the product that there are in the factors. $= 5170989$
~~~~~

**Step 3** Place the decimal point.  $= 51.70989$

**ANSWER**  $9.523 \times 5.43 = 51.70989$

b. **Step 1** Count the total number of decimal places in the factors.  $0.35 \times 0.2 = 70$   
3 decimal places

**Step 2** Count the same number of decimal places in the product that there are in the factors. Add zeros, if necessary.  $= 0070$   
~~~

Step 3 Place the decimal point. Drop any rightmost zeros. $= 0.07$

ANSWER $0.35 \times 0.2 = 0.07$

Using Zeros

Use rightmost zeros to help you place the decimal point correctly. Then drop any rightmost zeros in your final answer. So, 0.070 becomes 0.07.

EXAMPLE 2**Multiplying Decimals****Multiply.**

a. 0.3×0.6

b. 0.04×0.15

Solution**a. Step 1** Write the decimals in a column.

$$\begin{array}{r} 0.3 \\ \times 0.6 \\ \hline 18 \end{array}$$

Step 2 Multiply as you would whole numbers.

Step 3 Count the correct number of decimal places in the product. Place the decimal point. Add zeros to the left of the answer as needed.

0.18

Together, 0.3 and 0.6 have 2 decimal places.

ANSWER 0.18**CHECK** your answer by thinking about fractions:

$$0.3 \times 0.6 = 0.18, \text{ because } \frac{3}{10} \times \frac{6}{10} = \frac{18}{100}.$$

$$\begin{array}{r} \text{b. } 0.15 \\ \times 0.04 \\ \hline 00060 \end{array}$$

The factors have a total of 4 decimal places. Add zeros to the left of the answer so that the decimal point can be placed correctly.

ANSWER 0.006**CHECK** your answer by thinking about fractions:

$$0.15 \times 0.04 = 0.0060 \text{ or } 0.006, \text{ because } \frac{15}{100} \times \frac{4}{100} = \frac{60}{10,000} \text{ or } \frac{6}{1000}.$$

TRY THIS**Place the decimal point in the product.**

1. $0.48 \times 0.5 = 240$

2. $200.4 \times 10.26 = 2056104$

Multiply.

3. 0.7×0.08

4. 1.06×0.05

Summarize

Multiplying Decimals

- (1) Multiply the same way you would whole numbers.
- (2) Count the total number of decimal places in the factors.
- (3) Moving from the right to the left, count the same number of zeros in the product that there are in the factors. Add zeros on the left of the product if necessary.
- (4) Place the decimal point. Drop rightmost zeros in the final answer.

$$\begin{array}{r}
 0.05 \\
 \times 0.6 \\
 \hline
 0.030
 \end{array}$$

total of 3 decimal places

3 places

Add zeros.

ANSWER 0.03

Practice

Place the decimal point in the product.

- | | |
|------------------------------|---------------------------------|
| 1. $1.24 \times 0.08 = 992$ | 2. $7.18 \times 0.9 = 6462$ |
| 3. $5.26 \times 8.6 = 45236$ | 4. $0.003 \times 1.114 = 3342$ |
| 5. $1.4 \times 2.9 = 406$ | 6. $3.75 \times 0.6 = 2250$ |
| 7. $4.28 \times 0.05 = 2140$ | 8. $123.1 \times 2.45 = 301595$ |

Match the multiplication sentence with the product that has the decimal point in the correct place.

- | | |
|--|------------|
| 9. $85.4 \times 10.92 = 932568$ _____ | A. 9.32568 |
| 10. $8.54 \times 1.092 = 932568$ _____ | B. 93.2568 |
| 11. $85.4 \times 109.2 = 932568$ _____ | C. 932.568 |
| 12. $8.54 \times 10.92 = 932568$ _____ | D. 9325.68 |

Multiply.

- | | | |
|--------------------------|-------------------------|------------------------|
| 13. 0.14×0.2 | 14. 0.05×0.07 | 15. 0.4×0.009 |
| 16. 2.6×0.07 | 17. 18.5×0.2 | 18. 0.04×1.5 |
| 19. 0.24×2.4 | 20. 16.8×0.42 | 21. 36.03×1.3 |
| 22. 4.63×2.1 | 23. 5.08×0.36 | 24. 80.2×0.72 |
| 25. 412.9×0.011 | 26. 50.06×1.05 | 27. 9.344×7.8 |

Solve the problem. Explain your reasoning.

- 28.** A machine that bottles lotion can pour **15.6** ounces of lotion in one minute. How many ounces can the machine pour in **8.5** minutes?

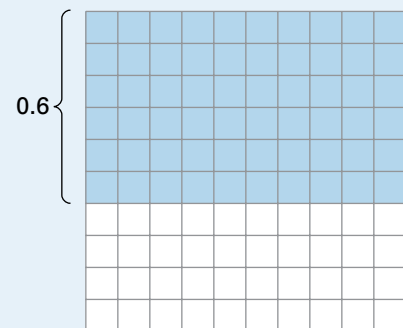
- 29.** Your kitten gains **0.6** pound in **1** month. At this rate, how much weight could the kitten gain in **4.25** months?

- 30.** You buy **0.75** pound of ground turkey priced at **\$7.76** a pound. How much do you pay for the package of turkey?

DID YOU GET IT?

- 31. Fill in the missing words.** The product of two decimals has the same number of _____ as the total number of _____ in the factors.

- 32. Use a model.** Use the model to multiply **0.6** by a decimal between **0** and **1** that has only one decimal place. Then write the product of the two decimals.



- 33. Explain your reasoning.** When you multiply the digits of **3.45** and **7.2** you get **24840**. How do you write your final answer? Explain.

LESSON
2-27

California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Divide Decimals by Whole Numbers

Getting Started You learned how to divide whole numbers by whole numbers. You can also divide decimals by whole numbers.

Dividing by Powers of Ten To divide a decimal by a power of ten, count the zeros in the power of ten. Then move the decimal point one place to the *left* for each zero. You may need to add zeros on the left of the answer to place the decimal correctly.

$$829.1 \div 100 = 8.291$$

There are 2 zeros.

EXAMPLE 1
Dividing Decimals by Powers of Ten

Divide: $9.457 \div 1000$

Solution

$$9.457 \div 1000 = 0.009457$$

There are 3 zeros.

$$= 0.009457$$

Count the zeros in the power of ten. There are 3. Count 3 place values to the left in the decimal. Add zeros as needed.

Place the decimal point three places to the left.

ANSWER 0.009457

TRY THIS

Divide.

1. $58.5 \div 100$

2. $467.72 \div 10$

Dividing Decimals by Whole Numbers To divide a decimal by a whole number, use long division. Bring the decimal point up from the dividend to the quotient, or answer.

$$\begin{array}{r}
 1.8 \leftarrow \text{quotient} \\
 \text{divisor} \rightarrow 2 \overline{) 3.6} \leftarrow \text{dividend} \\
 \underline{-2} \\
 16 \\
 \underline{-16} \\
 0
 \end{array}$$

EXAMPLE 2**Dividing Decimals by Whole Numbers**Divide: $4.8 \div 8$

Solution

$$\begin{array}{r} 0.6 \\ 8 \overline{)4.8} \end{array}$$

Divide using long division. Bring the decimal point up from the dividend to the quotient.

ANSWER 0.6

TRY THIS Divide.

3. $3.48 \div 3$

4. $3.927 \div 7$

Rounding a Decimal Quotient You may be asked to round your answer, or quotient, to the nearest tenth, hundredth, or thousandth. You learned how to round decimals in Lesson 2-19.

Using Zeros

If you are asked to round to the nearest hundredth but your answer only goes to the tenths' place, you do not need to add a 0 to the hundredths' place.

For example:

$$1.6 \div 8 = 0.2$$

Do not write 0.20 as the answer even if you were asked to round to the nearest hundredth.

EXAMPLE 3**Rounding a Decimal Quotient**Divide $2.45 \div 5$. Round to the nearest tenth.

Solution

$$\begin{array}{r} 0.49 \\ 5 \overline{)2.45} \\ -20 \\ \hline 45 \\ -45 \\ \hline 0 \end{array}$$

Divide using long division. Bring the decimal point up from the dividend to the quotient.

Round 0.49 to the nearest tenth. Since $9 > 5$, round 0.49 up to 0.5.

ANSWER 0.5

TRY THIS

Divide. Round to the nearest hundredth.

5. $0.86 \div 4$

6. $12.5 \div 6$

Divide. Round to the nearest thousandth.

7. $6.15 \div 8$

8. $2.75 \div 5$

Summarize**Dividing a Decimal by a Power of Ten**

Count the number of zeros in the power of ten. Move the decimal point one place to the left for each zero in the power of ten. Add zeros before the number if they are needed to place the decimal correctly.

$$658.7 \div 100 = 6.587$$

$$0.825 \div 100 = 0.00825$$

↑
zeros added

Dividing a Decimal by a Whole Number

Use long division. Bring the decimal point up in the answer.

$$1.4 \div 2 \quad \begin{array}{r} 0.7 \\ 2 \overline{)1.4} \\ \underline{-14} \\ 0 \end{array}$$

Practice

Divide.

1. $114.5 \div 100$

2. $9.7 \div 10$

3. $2478.5 \div 100$

4. $0.6 \div 100$

5. $526.4 \div 1000$

6. $449.76 \div 10$

7. $91.8 \div 10$

8. $0.987 \div 1000$

9. $24.2 \div 1000$

Match the division statement with the correct quotient.

10. $68.2 \div 2$ _____

A. 2.4

11. $654.1 \div 10$ _____

B. 0.7

12. $3.5 \div 5$ _____

C. 0.6541

13. $65.41 \div 100$ _____

D. 65.41

14. $43.2 \div 18$ _____

E. 34.1

Divide. Round to the nearest hundredth.

15. $0.64 \div 8$

16. $4.5 \div 15$

17. $0.4 \div 4$

18. $1.23 \div 3$

19. $0.072 \div 9$

20. $19.4 \div 1000$

21. $0.406 \div 100$

22. $2.78 \div 10$

23. $17.52 \div 12$

Divide. Round to the nearest thousandth.

24. $68.509 \div 7$

25. $57.56 \div 100$

26. $0.825 \div 10$

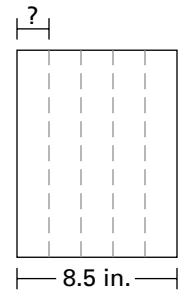
27. $1.4375 \div 3$

28. $504.57 \div 11$

29. $23.364 \div 6$

Solve the problem. Explain your reasoning.

- 30.** A sheet of paper is **8.5** inches wide. You fold the paper to divide it into **5** equal sections. What is the width of each section?



- 31.** A pack of **12** pencils costs **\$.96**. How much does **1** pencil cost?

- 32.** You place **10** equal-sized sugar cubes on a metric scale and record a measurement of **16.7** grams. What is the mass of one of these sugar cubes?

DID YOU GET IT?

- 33. Fill in the missing words.** When dividing a decimal by a whole number power of ten, move the decimal point one place to the _____ for each _____ in the power of ten.

- 34. Explain your reasoning.** Kayleigh uses the method shown at the right to divide $3.44 \div 2$. Is she correct? Why or why not?

| |
|-----------------------|
| 1.72 |
| 2 $\overline{) 3.44}$ |
| $\underline{-2}$ |
| 14 |
| $\underline{-14}$ |
| 04 |
| $\underline{-4}$ |
| 0 |

LESSON 2-28



California Standards

Gr. 5 NS 2.0: Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Divide by Decimals

Getting Started In Lesson 2-27, you learned how to divide decimals by whole numbers. You can also divide whole numbers by decimals and decimals by decimals.

Dividing by Decimals To divide by a decimal, change the decimal to a whole number. Do this by multiplying both the divisor and the dividend by a power of ten. This works because a division statement is also a fraction. Multiplying by the same number creates an equivalent fraction.

$$7.02 \div 5.4 = \frac{7.02}{5.4} \times \frac{10}{10} = \frac{70.2}{54} \text{ so } 5.4 \overline{)7.02} \text{ is the same as } 54 \overline{)70.2}.$$

EXAMPLE 1

Dividing by a Decimal

Divide.

a. $14.95 \div 6.5$

b. $1.68 \div 0.42$

Solution

a. **Step 1** Count the decimal places in the divisor. There is 1 decimal place, so you will multiply by 10.

$$14.95 \div 6.5$$

1 decimal place

Step 2 Multiply the divisor and dividend by 10.

$$14.95 \div 6.5 = 149.5 \div 65$$

Multiply by 10.

Step 3 Divide. Bring the decimal point up in the answer.

$$\begin{array}{r} 2.3 \\ 65 \overline{)149.5} \\ \underline{130} \\ 195 \\ \underline{195} \\ 0 \end{array}$$

b. **Step 1** Count the decimal places in the divisor. There are 2 decimal places, so you will multiply by 100.

$$1.68 \div 0.42$$

2 decimal places

Step 2 Multiply the divisor and dividend by 100. Drop the leftmost zero in the divisor.

$$1.68 \div 0.42 = 168 \div 42$$

Multiply by 100.

Step 3 Divide. Bring the decimal point up in the answer.

$$\begin{array}{r} 4 \\ 42 \overline{)168} \\ \underline{168} \\ 0 \end{array}$$

TRY THIS Divide.

1. $2.52 \div 1.8$

2. $0.72 \div 0.08$

Adding Zeros You will sometimes need to add zeros to the dividend when you multiply by a power of ten to make the divisor a whole number.

$$8.1 \div 0.09 = 8.10 \div 0.09 = 810 \div 9$$

Multiply by 100.

$$\begin{array}{r} 90 \\ 9 \overline{)810} \\ \underline{-81} \\ 0 \end{array}$$

EXAMPLE 2**Adding Zeros to Divide by a Decimal**

Divide. Round to the nearest tenth.

a. $0.7 \div 0.005$

b. $95 \div 1.6$

Solution

a. $0.7 \div 0.005$
3 decimal places

$$0.700 \div 0.005 = 700 \div 5$$

Multiply by 1000.

$$\begin{array}{r} 140 \\ 5 \overline{)700} \\ \underline{5} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Count the decimal places in the divisor. There are 3 decimal places, so you will multiply by 1000.

Add 2 zeros to the right of the dividend. Multiply the divisor and dividend by 1000. Drop the leftmost zeros in the dividend and the divisor.

Divide. You do not need to round this answer.

ANSWER $0.7 \div 0.005 = 140$

b. $95 \div 1.6$
1 decimal place

$$950 \div 1.6 = 950 \div 16$$

Multiply by 10.

$$\begin{array}{r} 59.37 \\ 16 \overline{)950.00} = 59.4 \\ \underline{80} \\ 150 \\ \underline{144} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 8 \end{array}$$

Count the decimal places in the divisor. There is 1 decimal place, so you will multiply by 10.

Add 1 zero to the right of the dividend. Multiply the divisor and dividend by 10.

Divide. Because you should round to the nearest tenth, you can stop dividing when you reach the hundredths' place.

ANSWER To the nearest tenth, $95 \div 1.6 \approx 59.4$.

Whole Number Dividends

Notice that in part (b) you are dividing a whole number by a decimal. Even though the dividend is a whole number, you still need to multiply by a power of ten so that the divisor is a whole number.

TRY THIS

Divide. Round to the nearest hundredth.

3. $4.6 \div 0.23$

4. $5.6 \div 0.007$

5. $3 \div 0.008$

6. $85 \div 6.4$

Summarize

Dividing by a Decimal

Multiply both the divisor and the dividend by a power of ten so that the divisor is a whole number. Divide. Bring the decimal point up from the dividend to the quotient, or answer.

$$8.05 \div 1.4 \rightarrow 1.4 \overline{)8.05} \rightarrow 14 \overline{)80.5}$$

Multiply by 10.

Adding Zeros to Divide by a Decimal

When necessary, add zeros to the dividend when you are multiplying by a power of ten.

$$6 \div 0.15 \rightarrow 0.15 \overline{)6} \rightarrow 15 \overline{)600}$$

Add 2 zeros to 6 because it was multiplied by 100.

Practice

Match the equivalent division statements.

1. $6.96 \div 2.9$ _____

A. $29 \overline{)6960}$

2. $0.696 \div 0.029$ _____

B. $29 \overline{)69,600}$

3. $69.6 \div 0.29$ _____

C. $29 \overline{)69.6}$

4. $6.96 \div 0.0029$ _____

D. $29 \overline{)696}$

Divide. Round to the nearest tenth.

5. $1.05 \div 0.7$

6. $0.072 \div 0.09$

7. $1.96 \div 1.4$

8. $6.24 \div 2.08$

9. $13.75 \div 1.9$

10. $2.8 \div 2.3$

11. $0.245 \div 0.7$

12. $0.114 \div 0.06$

13. $4.12 \div 1.03$

14. $18.25 \div 3.65$

15. $3.96 \div 1.7$

16. $1.024 \div 3.04$

Divide.

17. $6 \div 2.4$

18. $1.2 \div 0.15$

19. $54 \div 1.2$

20. $36.5 \div 3.65$

21. $0.09 \div 0.018$

22. $5832 \div 9.72$

23. $12 \div 0.4$

24. $285.6 \div 4.08$

25. $279.6 \div 0.233$

26. $937.5 \div 6.25$

27. $14.4 \div 0.24$

28. $133.7 \div 3.82$

Solve the problem. Explain your reasoning.

- 29.** Tim is setting up the course for a 9-mile walk. He places a sign every 0.15 mile along the path. How many signs will Tim place?



- 30.** You have \$124.50 to divide equally among your employees. You give each employee \$12.45. How many employees do you have?

- 31.** You spent \$150.94 on 31.25 shares of stock. What was the cost of each share?

DID YOU GET IT?

- 32. Fill in the missing words.** When dividing by a decimal, you need to rewrite the division statement so that the _____ is a _____.

- 33. Find the error.** Maddox uses the method shown at the right to divide $6 \div 0.15$. Is he correct? Why or why not?

| |
|-----------------------|
| 0.40 |
| 15 $\overline{)6.00}$ |
| $\underline{-60}$ |
| 0 |

LESSON
2-29

California Standards

Gr. 4 NS 2.1: Estimate and compute the sum or difference of whole numbers and positive decimals to two places.

Gr. 3/4/5/6/7 MR 2.1: Use estimation to verify the reasonableness of calculated results.

Estimate with Decimals

Getting Started In Lessons 2-23, 2-26, and 2-28, you learned how to add, subtract, multiply, and divide with decimals. You can also estimate sums, differences, products, and quotients involving decimals.

Estimating with Decimals You have had practice rounding decimals. To estimate with decimals, the first step is often rounding each decimal to the nearest whole number.

EXAMPLE 1
Estimating a Sum or Difference

Estimate.

a. $3.29 + 1.18$

b. $18.34 - 7.87$

c. $2.7 + 5.98$

Solution

a. $3.29 + 1.18$

$$\begin{array}{r} \downarrow \quad \downarrow \\ 3 \quad + \quad 1 \end{array}$$

Round to the nearest whole number.

$$3 \quad + \quad 1 = 4$$

Add.

ANSWER 4

b. $18.34 - 7.87$

$$\begin{array}{r} \downarrow \quad \downarrow \\ 18 \quad - \quad 8 \end{array}$$

Round to the nearest whole number.

$$18 \quad - \quad 8 = 10$$

Subtract.

ANSWER 10

c. $2.7 + 5.98$

$$\begin{array}{r} \downarrow \quad \downarrow \\ 3 \quad + \quad 6 \end{array}$$

Round to the nearest whole number.

$$3 \quad + \quad 6 = 9$$

Add.

ANSWER 9

TRY THIS

Estimate the sum or difference.

1. $4.57 + 1.2$

2. $12.15 - 5.48$

Checking for Reasonableness One use of estimation is to check if an answer is reasonable. To check whether the answer is reasonable, first round each decimal to the nearest whole number. Then perform the given operation on the rounded values—add, subtract, multiply, or divide. Finally, compare the rounded answer to the given answer.

EXAMPLE 2**Estimating a Product**

Estimate to check the reasonableness of the product:

$$1.89 \times 6.13 = 11.5857$$

Solution

$$1.89 \times 6.13 = 11.5857$$

$$2 \times 6 = \underline{\quad}$$

Round to the nearest whole number.

$$2 \times 6 = 12$$

Multiply the rounded values.

12 is close to 11.5857.

Compare the rounded answer to the given answer. If they are close, the answer is reasonable. If they are not close, the answer is not reasonable.

ANSWER The answer is reasonable.

EXAMPLE 3**Estimating a Quotient**

Estimate to check the reasonableness of the quotient:

$$31.836 \div 3.79 = 10.2$$

Solution

$$31.83 \div 3.79 = 10.2$$

$$32 \div 4 = \underline{\quad}$$

Round to the nearest whole number.

$$32 \div 4 = 8$$

Divide.

8 is much less than 10.2.

Compare the two quotients. If they are close, the answer is reasonable. If they are not close, the answer is not reasonable.

ANSWER The answer is *not* reasonable.

TRY THIS

Estimate to check the reasonableness of the product or quotient.

3. $7.86 \times 3.1 = 24.366$

4. $30.06 \div 5.32 = 5.65$

Summarize

Estimating a Sum or Difference

Round each decimal to the nearest whole number. Then add or subtract the rounded values.

$$1.4 + 6.9 = \underline{\hspace{2cm}}$$

$$1 + 7 = 8$$

$$5.3 - 2.9 = \underline{\hspace{2cm}}$$

$$5 - 3 = 2$$

Estimating to Check for Reasonableness

Round each decimal to the nearest whole number. Then perform the given operation with the rounded values. Compare the rounded answer to the given answer. If the answers are close, the given answer is reasonable. If they are not close, the given answer is *not* reasonable.

$$1.8 \times 8.2 = 18.76$$

$$2 \times 8 = 16$$

18.76 is *not* reasonable.

$$8.295 \div 2.1 = 3.95$$

$$8 \div 2 = 4$$

3.95 is reasonable.

Practice

Match the given statement with its estimated answer.

1. $4.88 + 4.49 = \underline{\hspace{2cm}}$

A. 16

2. $19.58 - 3.78 = \underline{\hspace{2cm}}$

B. 9

3. $4.06 + 1.96 = \underline{\hspace{2cm}}$

C. 6

Estimate the sum or difference.

4. $3.25 + 8.06$

5. $0.69 + 2.9$

6. $16.54 - 12.36$

7. $8.58 - 3.12$

8. $57.95 + 14.95$

9. $16.79 - 4.5$

Estimate to check the reasonableness of the sum or difference.

10. $9.45 + 11.87 = 21.32$

11. $14.82 + 1.03 = 15.85$

12. $5.07 + 7.39 = 14.46$

13. $3.67 + 3.21 = 6.88$

14. $2.04 + 10.75 = 10.79$

15. $8.75 - 6.98 = 1.77$

Estimate to check the reasonableness of the product or quotient.

16. $4.71 \times 7.39 = 28.8069$

17. $2.38 \times 2.65 = 6.307$

18. $8.94 \times 1.99 = 17.7906$

19. $6.25 \times 6.67 = 47.9375$

20. $3.88 \times 7.31 = 21.1988$

21. $8.967 \div 2.94 = 3.05$

22. $15.47 \div 4.76 = 3.25$

23. $39.69 \div 2.02 = 25.65$

Solve the problem. Use estimation to prove that your answer is reasonable.

- 24.** You have \$15 to spend on drinks and snacks. Do you have enough to buy 3 packs of grape juice and 2 packs of Sweet-and-Salty Mix?

| Snacks | |
|---------------------|--------|
| Sweet-and-Salty Mix | \$3.95 |
| Pretzels | \$1.98 |
| Yogurt Raisins | \$4.34 |
| Drinks | |
| Grape Juice Pack | \$2.97 |
| Apple Juice Pack | \$1.89 |
| Tropical Pack | \$2.51 |

- 25.** Your mother gives you 40.4 yards of string to distribute evenly among the students in your science lab. You cut equal lengths that measure 5.05 yards each. Your mother assumes that you have 8 students in your science lab. Is she correct?

- 26.** You make 5.75 quarts of iced tea. You fill two 1.5-quart containers. Do you have enough iced tea left to fill a 0.5-quart container?

DID YOU GET IT?

- 27. Fill in the missing words.** To estimate a sum, first round each _____ to the nearest _____.
- 28. Explain your reasoning.** Roberto subtracts $12.78 - 5.94$ to get 6.84. Explain whether his answer is reasonable.

LESSON
2-30

California Standards

Gr. 2 NS 5.1: Solve problems using combinations of coins and bills.

Gr. 2 NS 5.2: Know and use the decimal notation and the dollar and cent symbols for money.

Also included:

Gr. 3/4/5/6/7 MR 1.1,
Gr. 3/4/5/6/7 MR 2.1

Solve Problems with Decimals

Strategies to Remember

Sometimes a word problem may have too much or too little information:

Sometimes the facts you are given are *more than you need* to solve a problem.

Other times necessary *facts may be missing*.

EXAMPLE 1

Solving Problems with Missing Information

If possible, solve the problem. If it is not possible to solve, tell what additional information is needed.

Nathaniel has \$50.25. He purchases 2 DVDs for \$24.95 and a CD for \$7.20.

- How much money does he have left?
- How much did each DVD cost?

Solution

- Step 1 Identify** the operations needed. To find out how much Nathaniel has left, you need to subtract twice.

Step 2 Calculate. Subtract the first purchase: $50.25 - 24.95 = 25.30$
Subtract the second purchase: $25.30 - 7.20 = 18.10$

Step 3 Check for reasonableness: $50 - 25 = 25$; $25 - 7 = 18$. The answer is reasonable.

ANSWER Nathaniel has \$18.10 left.

- You know that the total cost of the 2 DVDS is \$24.95, but you are given no information about each DVD individually. So, you don't have enough information to solve this part of the problem.

TRY THIS Fill in the missing information to solve the problem, if possible.

- Kathy has \$104.29 in the bank. She deposits \$25.60 on Tuesday and \$15.14 on Thursday. How much does she have in the bank on Saturday?

Step 1 You need to _____ twice.

Step 2 Calculate: $\square + \square = \square$ $\square + \square = \square$

Step 3 She has _____ in the bank after she makes the deposit on Thursday. The problem does not give enough information to determine the amount in the account on Saturday.

EXAMPLE 2**Solving Problems with Extra Information**

Solve the problem, if possible. Identify any missing or extra information.

A deli sells turkey for \$7.26 a pound. You buy 10 deli rolls and 3 bags of turkey, each weighing 0.25 pound. How much do you pay for the turkey?

Remember

\$ is the dollar sign.

¢ is the cents sign.

usually, the combination of dollars and cents is written as a decimal:

\$2.50 means 2 dollars and 50 cents.

Solution

Step 1 Identify the operations needed. Multiply 7.26 by 0.25 to find the cost of 1 bag. Multiply this result by 3 to find the total amount you pay.

Step 2 Calculate. Multiply to find the cost of 1 bag: $7.26 \times 0.25 = 1.815$
Multiply to find the cost of 3 bags: $1.815 \times 3 = 5.445$

Step 3 Check for reasonableness. It does not make sense to pay \$5.445. Round to the nearest hundredths place, or the nearest cent.

ANSWER You pay \$5.45 for the turkey. The information about buying deli rolls was extra. It was not needed to solve the problem.

EXAMPLE 3**Solving Problems Using Different Operations**

Solve the problem, if possible. Identify any missing or extra information.

A single container of vanilla yogurt costs \$.54. A grocery store sells a pack of 6 containers for \$3.75. How much profit does the store make on each six-pack?

Solution

Step 1 Identify the operations needed. Multiply 0.54 by 6 to find the cost of a six-pack. Then subtract this amount from \$3.75 to find the profit.

Step 2 Calculate. Multiply to find the true cost of 6 containers: $0.54 \times 6 = 3.24$
Subtract to find the store's profit: $3.75 - 3.24 = 0.51$

Step 3 Check for reasonableness by estimating. At \$.50, a pack of 6 yogurts costs \$3.00. If the store charges \$3.75, they make about \$.75 a pack. The actual answer should be a little lower, so \$.51 is reasonable.

ANSWER The store makes \$.51 on each six-pack of yogurt. All of the information given in the problem was necessary.

TRY THIS

Solve the problem, if possible. Explain your reasoning.

2. You work at the library for \$8.12 an hour. Last week you worked 2.5 hours every day for 5 days. How much did you earn? Which days did you work?

Summarize

Solving Word Problems

- (1) Identify the operation or operations needed.
- (2) Decide whether there is enough information given to solve the problem, or if extra information was provided.
- (3) Perform the calculations to solve the problem, if possible.
- (4) Check your answer for reasonableness.

Practice

Read the problem. Number the steps of the solution to put them in order.

1. A puppy weighs 1.13 pounds. The puppy gains 0.38 pound the first week and 0.44 pound the second week. How much does the puppy weigh after 2 weeks?

SOLUTION: ____ add: $1.51 + 0.44$ ____ add: $1.13 + 0.38$

2. A vendor sells raisin mix for \$.29 per ounce. You pay for the raisin mix with a \$10 bill and get \$5.94 in change. How many ounces did you buy?

SOLUTION: ____ subtract: $10 - 5.94$ ____ divide: $4.06 \div 0.29$

Identify the operation suggested by the phrase.

3. 5 pounds at \$1.99 per pound
4. 0.3 liters less than yesterday
5. 1 can at \$2.50 for 3 cans

6. Fill in the missing information to solve the problem, if possible. Identify any extra or missing information.

This morning, Gaby had \$40.50. She spent \$5.52 on lunch and gave \$2.75 to a friend. Tomorrow, she will be paid \$22.00 for yard clean-up. How much does Gaby have now? What is her hourly pay for yard clean-up?

- a. **Step 1** To find out how much Gaby has now, you need to _____ twice.

Step 2 Calculate: $40.50 - 5.52 = \square$ $\square - 2.75 = \square$

Step 3 Gaby has _____ now.

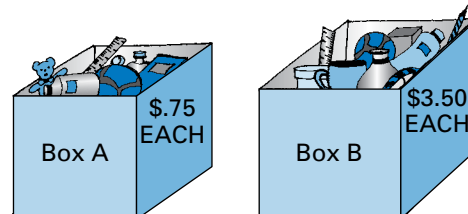
Step 4 The information about what Gaby will be paid tomorrow _____ (was/was not) needed to decide how much she has now.

- b. The amount Gaby will be paid tomorrow is _____. No information is given about the number of _____ she works. So, the hourly pay _____ (can/cannot) be determined.

First identify the operation(s) needed to find the answer. Then solve the problem, if possible. Explain your reasoning.

7. A 2.5-pound bag of rye flour costs \$1.95. A 4.5-pound bag of rye flour costs \$3.25. A 5-pound bag of white flour costs \$1.95. Which size bag of rye flour is the better buy?

8. At a garage sale, you mark two boxes as shown to the right. Someone buys 3 items from Box A and 2 items from Box B. How much do you earn from this sale?



9. Karl makes \$11.15 an hour at a karate school. In April, he worked a total of 8 days for 3.5 hours each day. During the month, he spent part of his earnings. How much did Karl have left at the end of the month?

DID YOU GET IT?

10. **Write a word problem.** Write a word problem using the order of the calculations at the right.

Step 1: $\$1.50 + \$3.40 = \$4.90$

Step 2: $\$4.90 - \$0.79 = \$4.11$

11. **Explain your reasoning.** Your friend says that the answer to the following problem is \$17.00. Is your friend's answer reasonable? Explain why or why not.

A store sells puzzle books for \$.85 each and pencils for \$.19 each. Jerry buys 1 pencil and 3 books. He gives the cashier \$20. How much change should he get?

Mixed Practice for Lessons 2-23 to 2-30

Vocabulary Review

Match the word with its definition.

Word

Definition

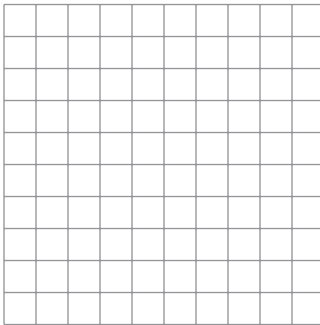
- | | | |
|-------------|-------|---|
| 1. dividend | _____ | A. a number that is to be divided |
| 2. product | _____ | B. an expression that represents a repeated factor |
| 3. power | _____ | C. a result of multiplying |

Fill in the missing word(s).

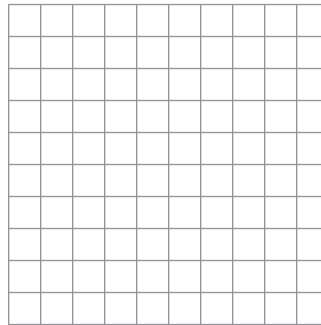
4. In the multiplication statement 5.89×0.4 , there are a total of three _____ in the factors.
5. To divide by a decimal, multiply the divisor and the dividend by a _____ so that the divisor is a _____.

Shade in the models below to find the product of the decimals.

6. 0.6×0.4 _____



7. 0.5×0.5 _____



Add, subtract, multiply, or divide.

- | | | |
|------------------------|--------------------|----------------------|
| 8. $3.36 \div 4$ | 9. $0.56 - 0.04$ | 10. $1.25 + 4.4$ |
| 11. 7.542×100 | 12. $1.487 + 0.3$ | 13. 0.009×2 |
| 14. $4.5 - 2.78$ | 15. $16.8 \div 16$ | 16. 6.009×8 |
| 17. $0.489 \div 10$ | 18. $2.9 + 7.972$ | 19. $21.492 - 6.51$ |

- 20.** Fill in the missing information to solve the problem.

Sam works at an internet catalog company. A batch of orders has a total file size of **575.52** kilobytes. Each order has a file size of **47.96** kilobytes. How many orders are in the batch?

Step 1 You are given the total file size of the batch and the size of each order in the batch. To find the number of orders in the batch, you need to _____.

Step 2 Calculate: $575.52 \div 47.96 =$ _____.

Step 3 There were _____ orders in the batch.

Step 4 My answer is reasonable because _____.

Add, subtract, multiply, or divide.

21. $58.43 + 12$

22. $9.7 - 5$

23. 0.8×0.14

24. $0.85 \div 1.7$

25. 3.48×5.7

26. $144.78 + 0.7$

27. $59.22 \div 6.3$

28. $0.98 - 0.0487$

Estimate the sum, difference, product, or quotient.

29. $27.3 - 7.4$

30. $35.7 \div 4.2$

31. $2.43 + 4.58$

32. 8.4×2.86

33. $9.44 \div 3.2$

34. 9.2×6.89

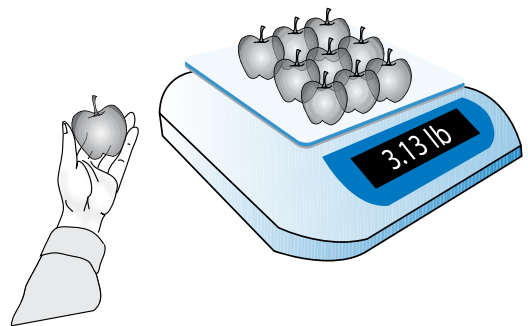
35. $3.62 + 7.43$

36. $5.7 - 2.847$

Solve the problem, if possible. Identify any extra or missing information.

Explain your reasoning.

- 37.** You place several apples on a digital scale. The scale reads **3.48 lb**. When you remove a large apple, the reading changes to the amount shown. How much does the large apple weigh? How much does the smallest apple weigh?



- 38.** Last week the manager at a printing company bought **200.65** ounces of ink at **\$1.40** an ounce. On average the printing company buys ink every **5** weeks. How much did the manager spend on the ink last week?

LESSON
2-31

California Standards

Gr. 4 NS 1.8: Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").

Gr. 5 NS 1.5: Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

Integers and the Number Line

Words to Remember

Integer: Any positive whole number, the opposite of any positive whole number, or zero

−18, −5, 0, 4, 122

Negative integer: Any integer that is less than 0

−357, −42, −3

Positive integer: Any integer greater than 0

1, 39, 422

Getting Started Previously you learned what whole numbers are and how to graph them on a number line. You can also place integers on a number line. On a number line positive integers are to the right of 0 and negative integers are to the left of 0.

EXAMPLE 1 Identifying Integers

Identify the integers in the following list.

3.3, −17, 22, $72\frac{1}{5}$

Solution

3.3 is not an integer since it contains a number after a decimal point.

−17 is an integer since it is the opposite of a positive whole number.

22 is an integer since it is a whole number.

$72\frac{1}{5}$ is not an integer since it contains a fraction, which is not a whole number.

TRY THIS

1. Circle the integers in the following list.

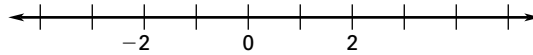
92, −100.66, $\frac{3}{8}$, 455, −672

2. Circle the negative integers in the following list.

12, −15, $\frac{5}{9}$, −2.5, −19, 28.75

EXAMPLE 2**Graphing Integers on a Number Line**

Place the integers -3 , 4 , 1 , and -2 on the number line.



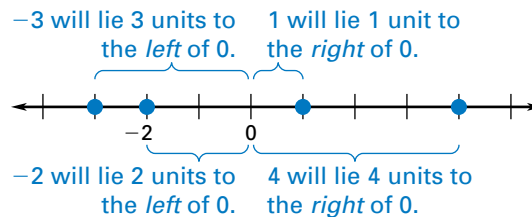
Solution

Step 1 **Determine** whether each number lies to the right or to the left of 0.

Negative integers lie to the *left* of 0: -3 and -2 .

Positive integers lie to the *right* of 0: 4 and 1 .

Step 2 **Place** each integer on the number line.

**EXAMPLE 3****Modeling a Situation**

The high temperature on Monday in San Francisco was 83°F .
The high temperature on Tuesday was 72°F . Represent the change in high temperatures from Monday to Tuesday using an integer.

Solution

Step 1 **Find** the change in temperature from Monday to Tuesday. $83 - 72 = 11^{\circ}\text{F}$

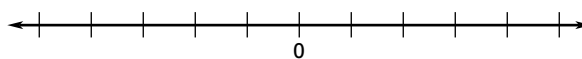
Step 2 **Determine** whether the temperature change should be positive or negative.

Since the temperature decreased, the sign should be *negative*.

ANSWER The high temperature change was -11°F .

TRY THIS

3. Place -1 , -4 , and 3 on the number line.



4. Manuel has a piece of ribbon **63** inches long.
After removing some of the ribbon for a project,
the ribbon is now **44** inches long. Use an integer
to represent the change in the length of the ribbon.



Summarize

An integer is a positive whole number, the opposite of a positive whole number, or zero.

Positive integers are positive whole numbers and are found to the right of zero on a number line.

Negative integers are opposites of positive whole numbers and are found to the left of zero on a number line.

Modeling Situations with Integers

Real-world situations can be modeled by integers. When the situation represents a decrease, use a negative integer to describe it. When the situation represents an increase, use a positive integer to describe it.

Practice

Match the description with the integer that represents it.

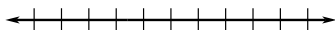
- | | |
|-----------------------------------|---------------|
| 1. An increase of 11 points _____ | A. -30 ft |
| 2. A decrease of 11 points _____ | B. 11 points |
| 3. A decrease of 30 feet _____ | C. -11 points |
| 4. An increase of 30 feet _____ | D. 30 ft |

Circle the integers.

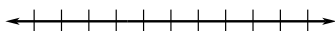
- | | |
|---|--|
| 5. $-\frac{4}{5}$, 99, 122.55, -37, 14 | 6. 100.1, 0.5, $45\frac{6}{7}$, -1055, 7 |
| 7. 676, -932.9, 12, $\frac{17}{43}$ | 8. -15, -68 $\frac{33}{51}$, 55.17, $\frac{3}{4}$ |
| 9. -722.1, -1434, $222\frac{3}{4}$, 0 | 10. 10,922, 3.46, -5 |

Place the integers on a number line.

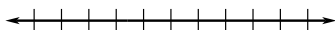
11. -1, 0, 2, -5



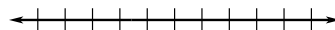
13. -2, 3, 5, 6



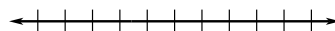
15. 12, 5, 0, -2



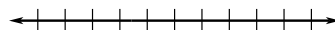
12. -6, -1, 1, -4



14. -10, 4, 6, -4



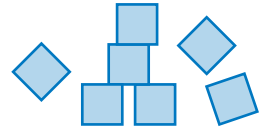
16. -13, 12, 8, -4



Tell if the situation represents an *increase* or a *decrease*. Then represent the amount of the increase or decrease by an integer.

- 17.** Karla has \$20 to spend at a jewelry store. After buying a bracelet she has \$6 remaining.

- 18.** Hannah is building a tower of blocks with her younger brother. They currently have 14 blocks stacked in their tower. After her younger brother knocks over the tower there are 4 blocks left standing.



- 19.** Jesse is filling an empty bucket with water. After he fills the bucket there are 3 gallons of water in it.

- 20.** You and a friend order a pizza that has 10 slices in it. After you and your friend finish eating, there are 4 slices remaining.

DID YOU GET IT?

- 21. Fill in the missing words.** To place integers on a number line, graph _____ to the right of zero and graph _____ to the left of zero.

- 22. Explain your reasoning.** What kinds of numbers are integers?

- 23. Describe.** Give an example of a real-life increase and an example of a real-life decrease that can be described by integers. Explain your thinking.

LESSON
2-32

California
Standards

Gr. 7 NS 2.5: Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra 1.2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

Opposites and Absolute Value

Words to Remember

Opposites: Two numbers that are the same distance from **0** on a number line, but on opposite sides of **0**

7 and -7 are opposites.

9 and -9 are opposites.

Absolute value of a number: The distance a number is from **0** on a number line.

Remember that distance is always a positive quantity (or zero). Show absolute value with vertical bars on each side of the number, for example $|17|$.

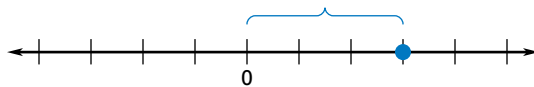
Getting Started You learned how to place integers on a number line. Now you will find the distance between a point and zero on the number line to find the *opposite* of a number or the *absolute value* of a number.

EXAMPLE 1 Finding the Opposite of an Integer

Use a number line to find the opposite of 3.

Solution

Step 1 Place 3 on a number line.



Step 2 Determine how far 3 is from 0. 3 is 3 units to the right of 0.

Step 3 Find the opposite of 3. The opposite of a number must be the same distance from **0** as the original number, but in the opposite direction. So, the opposite of 3 must be 3 units to the left of **0**, or at **-3** .

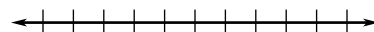
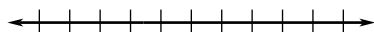
ANSWER The opposite of 3 is **-3** .

TRY THIS Use a number line to find the opposite of the number.

1. 4



2. -2



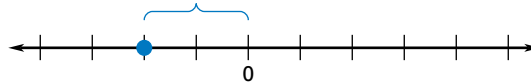
EXAMPLE 2**Finding the Absolute Value of an Integer**

Use a number line to find $|-2|$

Solution

The absolute value of a number is the distance the number is from 0.

Step 1 Graph -2 on a number line.



Step 2 Determine how far -2 is from 0. -2 is 2 units to the left of 0.

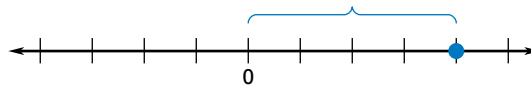
ANSWER The absolute value of -2 must be 2 because distances are always positive (or 0).

EXAMPLE 3**Finding the Opposite and Absolute Value of an Integer**

Use a number line to find the opposite of 4 and to find $|4|$.

Solution

Step 1 Place 4 on a number line.



Step 2 Determine how far 4 is from 0. 4 is 4 units to the right of 0.

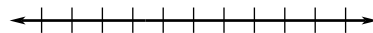
Step 3 The opposite of 4 must be the same distance from 0 but in the opposite direction. So, the opposite of 4 is -4 . The absolute value of 4 must be 4 because distances are always positive (or 0).

ANSWER The opposite of 4 is -4 , and $|4| = 4$.

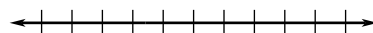
TRY THIS

Use a number line to evaluate the expression.

3. $|7| = \square$



4. The opposite of $-6 = \square$ and $|-6| = \square$.



Summarize

Opposite

The *opposite* of a number is a number that is the same distance from zero on a number line as the given number, but on the opposite side of zero.

Absolute Value

The *absolute value* of a number is the distance between 0 and the number on a number line. Remember that distance is always a positive quantity (or zero). Absolute value is shown by vertical bars on each side of the number.

Practice

Match the description with the number that represents it. You may use some answer choices more than once or not at all.

- | | | |
|--------------------|-------|--------|
| 1. Opposite of 19 | _____ | A. -19 |
| 2. $ 91 $ | _____ | B. 19 |
| 3. $ -19 $ | _____ | C. -91 |
| 4. Opposite of -91 | _____ | D. 91 |

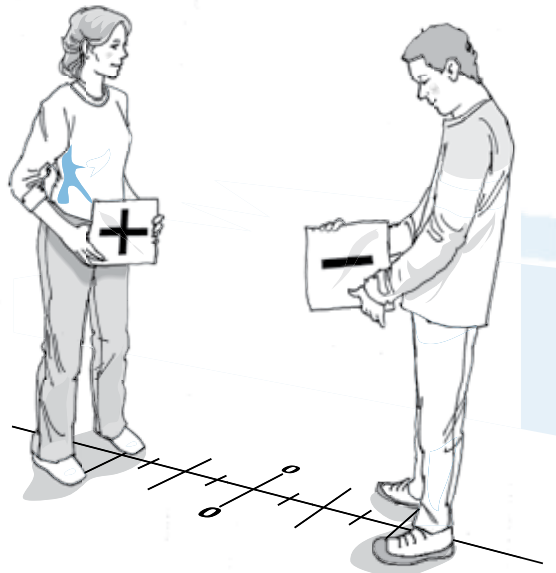
Evaluate the expression.

- | | | | | | |
|---------------------|--------------------------|---------------------|--------------------------|---------------------|--------------------------|
| 5. Opposite of -7 | <input type="checkbox"/> | 6. Opposite of 6 | <input type="checkbox"/> | 7. $ -9 $ | <input type="checkbox"/> |
| 8. Opposite of -2 | <input type="checkbox"/> | 9. $ 1 $ | <input type="checkbox"/> | 10. Opposite of 28 | <input type="checkbox"/> |
| 11. Opposite of 44 | <input type="checkbox"/> | 12. $ -199 $ | <input type="checkbox"/> | 13. Opposite of -50 | <input type="checkbox"/> |
| 14. $ 0 $ | <input type="checkbox"/> | 15. $ -762 $ | <input type="checkbox"/> | 16. Opposite of 10 | <input type="checkbox"/> |
| 17. $ 78 $ | <input type="checkbox"/> | 18. Opposite of 92 | <input type="checkbox"/> | 19. Opposite of -31 | <input type="checkbox"/> |
| 20. Opposite of -74 | <input type="checkbox"/> | 21. Opposite of 936 | <input type="checkbox"/> | 22. $ -302 $ | <input type="checkbox"/> |
| 23. $ -4002 $ | <input type="checkbox"/> | 24. Opposite of 76 | <input type="checkbox"/> | 25. $- 668 $ | <input type="checkbox"/> |
| 26. Opposite of 65 | <input type="checkbox"/> | 27. Opposite of -32 | <input type="checkbox"/> | 28. $- -8701 $ | <input type="checkbox"/> |

Read and solve the problem. Then explain your answer.

- 29.** Simone told you that she placed a point on a number line that had an absolute value of 15. At what two numbers could she have graphed her point?

- 30.** Julia and Roberto are each holding cards with integers on them. One has a positive integer and the other has a negative integer. The teacher has put a number line on the floor using tape and asks Julia and Roberto to stand on the points that represent their numbers. How can they determine if their numbers are opposites without showing each other their cards?



DID YOU GET IT?

- 31. Fill in the missing words.** The _____ of a number is the distance between _____ and the number on a number line.
- 32. Explain your reasoning.** What is the opposite of the opposite of -3 ?
- 33. Explain your reasoning.** How are the opposite of a number and the absolute value of a number alike? How are they different?

LESSON
2-33

California
Standards

Gr. 5 NS 1.5: Identify and represent on a number line decimals, fractions, mixed numbers, and **positive and negative integers**.

Gr. 6 NS 1.0: Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.

Compare and Order Integers

Words to Remember

Compare: To relate two or more numbers based on their sizes

$$-9 < 0; -4 > -5$$

Order: To write a group of numbers in a particular way based on their sizes

From least to greatest, the numbers -3 , 6 , and -12 are -12 , -3 , and 6 .

Getting Started You learned how to find the absolute value of an integer. Now you will learn how to order several integers based on their values.

EXAMPLE 1

Comparing Integers

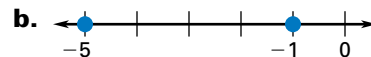
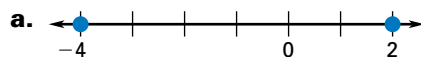
Complete the statements with $<$, $>$, or $=$.

a. -4  2

b. -1  -5

Solution

Step 1 Place the points on a number line.



Step 2 Find the integers farthest to the right.

a. 2 is greater than -4

b. -1 is greater than -5

Step 3 Insert the correct inequality sign.

The inequality symbol should open towards the greater integer.

a. -4  2

b. -1  -5

TRY THIS

Complete the statement with $<$, $>$, or $=$.

1. 4  -4

2. -7  0

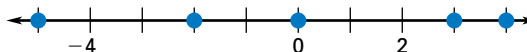
EXAMPLE 2**Ordering Integers Using a Number Line**

Use a number line to order the integers from least to greatest.

4, -5, 3, 0, -2

Solution

Step 1 Place the points on a number line.



Step 2 Order the integers. Least to greatest is left to right.

-5, -2, 0, 3, 4

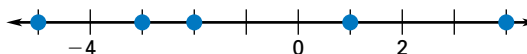
EXAMPLE 3**Ordering Integers**

Order the integers from greatest to least.

-3, -5, 1, 4, -2

Solution

Step 1 Place the points on a number line.



Step 2 Order the integers. Greatest to least is right to left.

4, 1, -2, -3, -5

TRY THIS

Order the integers.

3. Least to greatest: 3, -7, 8, -1

4. Greatest to least: -6, 8, 4, -3

Summarize

Ordering Integers from Least to Greatest

Begin by placing the integers on a number line. Then order the points as they appear on the number line. From *left to right* is the order of the integers from *least to greatest*.

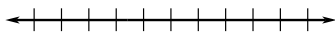
Ordering Integers from Greatest to Least

Begin by placing the numbers on a number line. Then order the points as they appear on the number line. From *right to left* is the order of the integers from *greatest to least*.

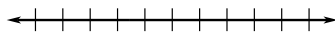
Practice

Use the number line to order the integers from greatest to least.

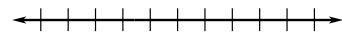
1. 9, -7, 4, 1, -2



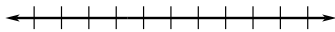
2. 3, 10, -7, 6, -8



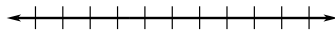
3. -12, -6, 4, 0, 1



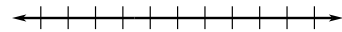
4. 10, 5, -3, -5, 11



5. -2, -14, -8, -1, -6

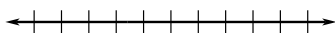


6. 0, -13, -25, -11, -22

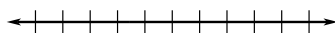


Use the number line to order the integers from least to greatest.

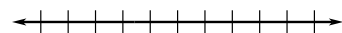
7. 8, -2, 0, -4, 3



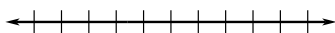
8. -3, 4, 10, -10, 0



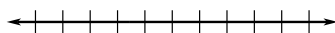
9. 8, -8, 0, 7, -7



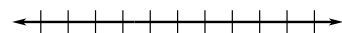
10. -4, -2, -10, 8, 4



11. 0, -15, -22, 19, -14



12. -18, -20, -2, -30, -13

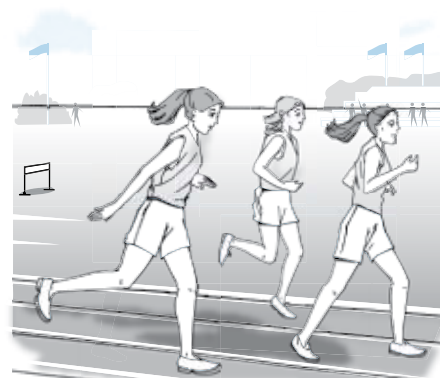


Read the problem and answer the questions.

- 13.** Morgan and Mackenzie are comparing their hair length to their friend Jenna's hair length. Morgan states that her hair is $+3$ inches compared to Jenna's hair and Mackenzie states that her hair is -2 inches compared to Jenna's hair. Who has the shortest hair? Write the girls' names in order of their hair length from shortest to longest.

- 14.** The average temperatures during the winter months in a city are -12°F , 10°F , -2°F , 5°F , and -3°F . What is the order of the temperatures from greatest to least?

- 15.** Paige, Amber, and Bailey are running around a track to see how their times compare with the school record. Paige ran -2 seconds as compared to the record, Amber ran $+6$ seconds as compared to the school record, and Bailey ran $+4$ seconds as compared to the school record. Who had the fastest time? Write the girls' names in order from fastest to slowest.



DID YOU GET IT?

- 16. Fill in the missing words.** To order integers from greatest to least using a number line, the greatest integer appears farthest to the _____ and the least integer appears farthest to the _____.
- 17. Compare.** How can you compare a positive integer to a negative integer?

LESSON 2-34



California Standards

Gr. 6 NS 1.1: Compare and order positive and negative fractions, decimals, and mixed numbers and **place them on a number line**.

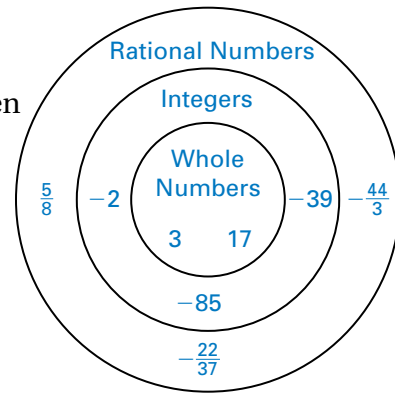
Gr. 7 NS 1.5: Know that **every rational number is either a terminating or repeating decimal** and be able to convert terminating decimals into reduced fractions.

Rational Numbers

Words to Remember

Rational number: Any number that can be written as the quotient of two integers when the denominator is not zero. The rational numbers include integers, positive and negative fractions and mixed numbers, and positive and negative terminating or repeating decimals.

Decimals that do not terminate or repeat are not rational numbers.



Getting Started You identified and placed integers on a number line. Now you will identify and graph rational numbers on a number line.

Look Ahead

In a later course you will learn how to write repeating decimals as quotients.

$$0.\bar{3} = \frac{1}{3}, \text{ so}$$

$$3.\bar{3} = 3\frac{1}{3} = \frac{10}{3}.$$

$3.\bar{3}$ is a rational number.

EXAMPLE 1 Writing Integers as Quotients

Show that -3 is a rational number.

Solution

Write -3 as a quotient of integers.

$$-3 = -3 \div 1 = \frac{-3}{1} = -\frac{3}{1} \checkmark$$

EXAMPLE 2 Identifying Rational Numbers

Which numbers listed below are rational numbers?

-22 , 0.2 , $3.4927\dots$, $3.\bar{3}$

Solution

-22 is a rational number. It can be written as $-\frac{22}{1}$.

0.2 is a rational number. It can be written as $\frac{2}{10} = \frac{1}{5}$.

$3.4927\dots$ is not a rational number. The decimal does not terminate or repeat.

$3.\bar{3}$ is a rational number. It is a repeating decimal.

TRY THIS Circle the rational numbers.

1. 58, 1.77, 3.303003 ... 2. -12.1 , 0 , $5.\bar{6}$

EXAMPLE 3 Graphing Rational Numbers on a Number Line

Place the rational numbers on a number line.

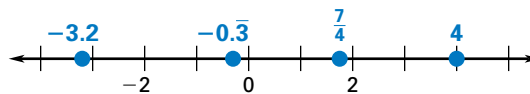
$$-3.2, 4, \frac{7}{4}, -0.\bar{3}$$

Solution**Step 1** Identify any numbers that are not integers. Determine between which two integers they fall. -3.2 falls between -3 and -4 . 4 will be placed on 4 .

$$\frac{7}{4} = 1.75 \text{ and falls between } 1 \text{ and } 2.$$

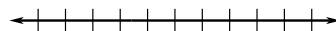
 $-0.\bar{3}$ falls between 0 and -1 .**Step 2** Determine approximately where between the integers you found in Step 1 the rational number should be placed. Then graph the number. -3.2 lies closer to -3 than to -4 . 4 should be placed on 4 .

$$\frac{7}{4} = 1.75, \text{ which lies closer to } 2 \text{ than to } 1.$$

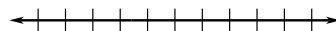
 $-0.\bar{3}$ lies closer to 0 than to -1 .**TRY THIS** Graph the rational numbers on a number line.

3. $\frac{6}{5}, -2.4, 3\frac{1}{2}$

$$\frac{6}{5} = \square, \text{ which lies closer to } \square \text{ than to } \square.$$

 -2.4 lies closer to \square than to \square . $3\frac{1}{2}$ lies between \square and \square .

4. $-3.\bar{6}, -1, \frac{8}{3}$



Summarize

A *rational number* is an integer, or the quotient of any two integers when the denominator is not zero. Any decimal that either terminates or repeats is a rational number.

Placing Rational Numbers on a Number Line

When the rational number is not an integer, first determine between which two integers the number falls. Next determine where between the two integers it should be placed and then graph it on the number line.

Practice

Match each decimal with its description.

1. -42.455 _____

A. Terminates

2. $6.3471 \dots$ _____

B. Repeats

3. $8.\overline{04}$ _____

C. Neither terminates nor repeats

Tell whether or not the given number is *rational* (R) or *not rational* (N).

4. -52 _____

5. 4.135 _____

6. $2.397 \dots$ _____

7. $\frac{14}{5}$ _____

8. $\frac{8}{9}$ _____

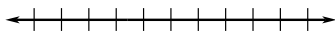
9. $43.097\overline{4}$ _____

10. 3.21452 _____

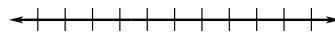
11. $6.141141114 \dots$ _____

Place the rational numbers on a number line.

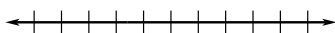
12. $\frac{3}{5}, -2.5, 4.9, -1$



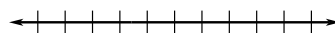
13. $\frac{19}{5}, 1.62, -\frac{14}{3}, -0.5$



14. $-4\frac{1}{5}, -0.22, -1.\overline{4}, 1.4$

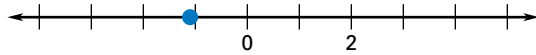


15. $\frac{14}{3}, -1.9, -0.\overline{7}, -3$



Determine whether the student's work is correct. Explain your answer.
If the student is incorrect, correct the mistake.

- 16.** Shaunda has graphed -0.9 as shown.



- 17.** Korey states that $7.898898889 \dots$ is a rational number because it is a repeating decimal.

- 18.** Mya states that all whole numbers are rational numbers.

- 19.** Dillan states the numbers -2 , $\frac{7}{8}$, 0.8 , $1\frac{1}{3}$, 2.1 are all examples of rational numbers.

DID YOU GET IT?

- 20. Fill in the missing words.** Any decimal that is either _____ or _____ is a rational number.

- 21. Explain your reasoning.** How are whole numbers, integers, and rational numbers related?

LESSON
2-35

California
Standards

Gr. 5 NS 1.5: Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

Gr. 6 NS 1.1: Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

Compare and Order Rational Numbers

Words to Remember

Common denominator: When two fractions are written with the same denominator, the denominator is called a *common denominator*

$$\begin{array}{r} \frac{2}{7} \frac{5}{7} \\ \frac{3}{15} \frac{10}{15} \end{array} \quad \begin{array}{l} \leftarrow \text{Common} \\ \text{denominator} \end{array}$$

Getting Started In Lesson 2-33 you compared and ordered integers. Now you are going to compare and order rational numbers.

Comparing Rational Numbers There are several methods for comparing and ordering rational numbers. When the rational numbers have a *common denominator*, compare the numerators. When the rational numbers do not have common denominators, rewrite them so they have common denominators, change them to decimals, or place them on a number line to make the comparison.

EXAMPLE 1

Comparing Two Rational Numbers

Fill in the circle with $<$, $>$, or $=$.

$$\frac{18}{5} \quad \bullet \quad 3\frac{3}{4}$$

Solution

$$3\frac{3}{4} = \frac{15}{4}$$

Write as an improper fraction.

$$\frac{18}{5} = 3.6 \text{ and } \frac{15}{4} = 3.75$$

Since denominators are not the same, change to decimals for comparison.

$$3.6 < 3.75, \text{ so } \frac{18}{5} < 3\frac{3}{4}.$$

Answer

Another Way

In Example 1, you can write the two fractions with a common denominator of 20 instead. Then compare their numerators.

TRY THIS

Fill in the circle with $<$, $>$, or $=$.

1. $12.45 \quad \bullet \quad \frac{249}{20}$

2. $-0.22 \quad \bullet \quad -\frac{6}{25}$

EXAMPLE 2**Ordering Rational Numbers**

Use a number line to order the rational numbers from greatest to least.

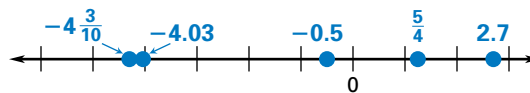
$$2.7, -4\frac{3}{10}, -0.5, \frac{5}{4}, -4.03$$

Solution

Step 1 Convert the fractions to decimals so you can place them easily on a number line.

$$-4\frac{3}{10} = -4.3, \frac{5}{4} = 1.25$$

Step 2 Place and label the points on a number line.



Step 3 Write the numbers as they are placed on the number line from right to left.

ANSWER From greatest to least, the numbers in order are:

$$2.7, \frac{5}{4}, -0.5, -4.03, -4\frac{3}{10}$$

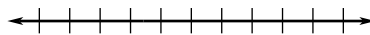
Remember

The greatest numbers are the farthest to the right on the number line and the least numbers are the farthest to the left on the number line.

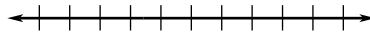
TRY THIS

Use a number line to order the numbers from greatest to least.

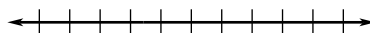
3. $2.43, -2\frac{7}{10}, -\frac{19}{5}, 1.07, 0.12$



4. $3\frac{7}{20}, -0.8, \frac{17}{3}, -3.82, -\frac{7}{2}$



5. $-\frac{5}{4}, -0.2, 4.31, -3, \frac{5}{2}, -\frac{13}{3}$



Summarize

Comparing Rational Numbers

Write the rational numbers with a common denominator and then compare numerators, or convert them to decimals and compare them.


Ordering Rational Numbers

Convert the rational numbers all to the same form, for example decimals. Then compare the decimals, or place the numbers on a number line and then write the order they appear on the number line. The greatest numbers are farthest to the right and the least numbers are farthest to the left.

Practice

Fill in the circle with $<$, $>$, or $=$.

1. -6.5  $4\frac{1}{8}$

3. $\frac{89}{20}$  $\frac{9}{2}$

5. -48.875  $-\frac{391}{8}$

2. $5\frac{9}{25}$  5.34

4. $\frac{503}{100}$  $50\frac{3}{100}$

6. $\frac{3}{5}$  0.501

Order the numbers from greatest to least.

7. -4.2 , $3\frac{7}{8}$, $-4\frac{2}{5}$, 2.08

8. 3.06 , 4.1 , $-\frac{23}{3}$, -8.9

9. -0.97 , $-\frac{907}{1000}$, $-\frac{9}{10}$

10. $104\frac{3}{16}$, -10.998 , $\frac{1049}{10}$, -10.99

Order the numbers from least to greatest.

11. 2.01 , $\frac{21}{10}$, -3.66 , $-3\frac{2}{3}$

12. $-4\frac{1}{100}$, 4.101 , -4.101 , $4\frac{1}{100}$

13. $\frac{26}{3}$, $\frac{17}{6}$, $\frac{33}{12}$, $\frac{27}{4}$

14. -13.25 , $-13\frac{2}{5}$, -12.9 , -13.09

- 15.** Carlos needs a piece of wood for a shelf that is **13.25** feet long. He found one that is $13\frac{3}{20}$ feet long and another one that is $\frac{133}{10}$ feet long. Is either piece of wood long enough for his shelf? If so, which one? If not, how much too short are the pieces he found? Explain.

- 16.** Four members of a model airplane club have brought their model airplanes, of different sizes, to a meeting. The weight of Garrett's airplane is **12.14** pounds, Keisha's airplane weighs $12\frac{11}{25}$ pounds, Tyrei's airplane weighs **10.9** pounds, and the weight of Samuel's airplane is $\frac{59}{5}$ pounds. List the students' names in order of their airplane weights, from least to greatest. Explain your reasoning.



- 17.** You have a rope that is **4** feet long. Your friend has a rope that is $1\frac{2}{3}$ yards long. Who has the longer rope? Explain your reasoning.

DID YOU GET IT?

- 18. Fill in the missing words.** When ordering rational numbers using a number line, the greatest numbers are farthest to the _____ and the least numbers are farthest to the _____.
- 19. Describe a process.** Describe two methods you can use to compare and order rational numbers.

Mixed Practice for Lessons 2-31 to 2-35

Vocabulary Review

Match the word with its mathematical meaning and its everyday meaning.

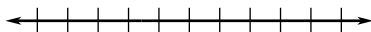
| Word | Mathematical meaning | Everyday meaning |
|-------------------------|--|------------------------------------|
| 1. opposite ____ , ____ | A. any number that can be written in the form $\frac{a}{b}$, where $b \neq 0$ | X. an instruction to do something |
| 2. rational ____ , ____ | B. to compare a set of numbers and sort them by value | Y. facing or across from something |
| 3. order ____ , ____ | C. a number that is the same distance from zero on a number line as a given number, but in a different direction | Z. clear or sensible thinking |

Fill in the word.

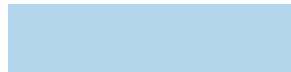
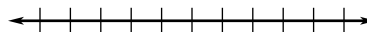
4. The _____ of a number is the distance the number is from zero on a number line.

Place the numbers on the number line to order the numbers from least to greatest.

5. $-5, 2, 4, 0, -3$



6. $1.4, -\frac{3}{4}, 3\frac{1}{5}, -2.7$



Find the opposite and the absolute value of the number.

7. 16 ,

8. -81 ,

9. -28 ,

10. 72 ,

Tell whether the number is *rational* or *not rational* by circling the correct word(s).

11. $0.4239\dots$ rational, not rational

12. 0.689 rational, not rational

13. -4 rational, not rational

14. $3.\overline{75}$ rational, not rational

15. $\frac{5}{6}$ rational, not rational

16. $-\frac{1}{12}$ rational, not rational

Write the number as a rational number in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

17. $5.1 = \frac{\boxed{}}{\boxed{}}$

18. $3.3 = \frac{\boxed{}}{\boxed{}}$

19. $-0.4 = \frac{\boxed{}}{\boxed{}}$

20. $6.88 = \frac{\boxed{}}{\boxed{}}$

21. $0.7 = \frac{\boxed{}}{\boxed{}}$

22. $5.33 = \frac{\boxed{}}{\boxed{}}$

23. $6.4 = \frac{\boxed{}}{\boxed{}}$

24. $7.908 = \frac{\boxed{}}{\boxed{}}$

25. In golf or miniature golf, *par* is the number of strokes needed by an expert to complete a course. Golf scores can be stated as positive or negative integers to show whether a player's score is above or below par. Eli and four friends play miniature golf. The final scores are Alan: 4, Marty: 0, Eli: -3, Cade: -1, and Ori: 1. Order the scores from least to greatest. Who won the game?



26. On Macy's monthly bank account statement, a deposit of money is represented by a positive number and a withdrawal of money is represented by a negative number. One month her bank statements showed these transactions:

$-25, 57.75, 23, -42.15, 24.90, 15.55, -80.01$

What was Macy's greatest withdrawal? What was her greatest deposit?

ACTIVITY 2-36



California Standards

Gr. 6/7 MR 1.2: Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posted.

Gr. 7 MR 2.4: Make and test conjectures by using both inductive and deductive reasoning.

Also included: Gr. K MR 2.1, Gr. 3/4/5/6 MR 3.0, Gr. 7 MR 3.0

Adding Integers Using Models

Goal: Use concrete objects to understand how to add integers

Materials: +1 and -1 cards.

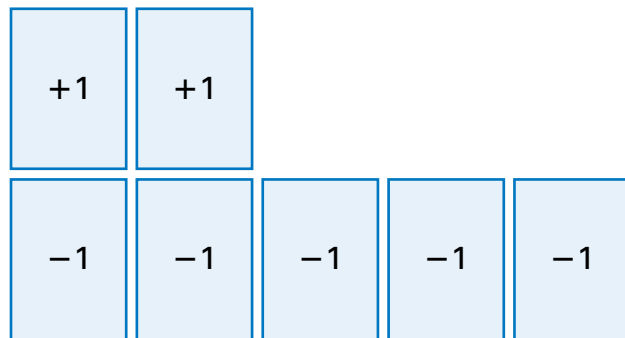
Getting Started You can use +1 and -1 cards to model integer addition. One +1 card and one -1 card combine to make zero. This is called a *zero pair*.

$$\begin{array}{|c|} \hline +1 \\ \hline \end{array} \begin{array}{|c|} \hline -1 \\ \hline \end{array} = 1 + (-1) = 0$$

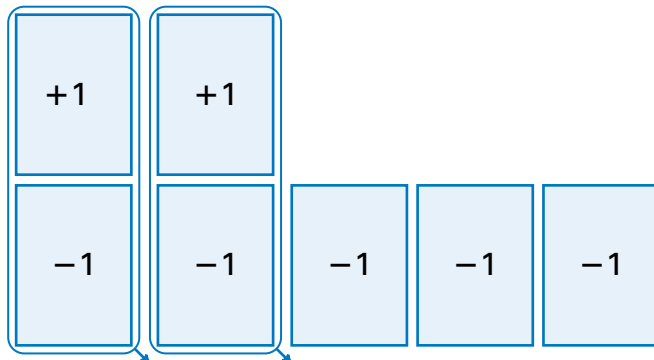
EXAMPLE

Use the following steps to model $2 + (-5)$ with +1 and -1 cards.

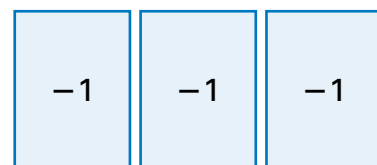
Step 1 Model $2 + (-5)$.



Step 2 Group and remove any zero pairs.



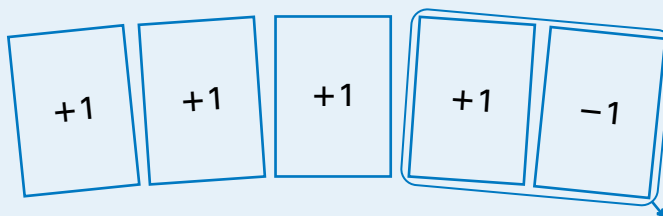
Step 3 Write the result.



ANSWER Since there are 3 (-1) cards remaining, $2 + (-5) = -3$.

MAKE IT A GAME!

- Form groups of 3 or 4 students, and choose a dealer.
- The dealer gives 5 cards to each person in the group.
- Each person writes the integer addition problem for their hand and finds the sum.
- The person with the greatest sum scores 1 point.
- The players return their cards, and the dealer shuffles and deals again.
- The first person to score 3 points wins the game.



$4 + (-1) = 3$,
so the value of
this hand is 3.

Practice

Use +1 and -1 cards to find the sum of the positive and negative integers.

1. $5 + (-7)$ 2. $3 + (-2)$ 3. $3 + (-1)$ 4. $1 + (-5)$

5. **Make a Conjecture** Based on your answers to Exercises 1–4, make a conjecture about how to find the sum of a positive and a negative integer without using +1 and -1 cards.

Use your conjecture in Exercise 5 to find the sum.

6. $4 + (-3)$ 7. $6 + (-2)$ 8. $2 + (-8)$ 9. $4 + (-7)$

Use +1 and -1 cards to find the sum of the two negative integers.

10. $-1 + (-2)$ 11. $-2 + (-2)$ 12. $-3 + (-5)$ 13. $-3 + (-1)$

14. **Make a Conjecture** Based on your answers to Exercises 10–13, make a conjecture about how to find the sum of two negative integers without using +1 and -1 cards.

Use your conjecture in Exercise 14 to find the sum.

15. $-5 + (-1)$ 16. $-4 + (-2)$ 17. $-8 + (-4)$ 18. $-20 + (-10)$

DID YOU GET IT?

Find the sum of the integers.

19. $-5 + (-3)$ 20. $2 + (-8)$ 21. $-7 + 5$ 22. $-9 + (-11)$

LESSON 2-37



California Standards

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; **add with negative integers;** subtract positive integers from negative integers; **and verify the reasonableness of the results.**

Gr. 6 NS 2.3: Solve **addition**, subtraction, multiplication, and division **problems**, including those arising in concrete situations, **that use positive and negative integers** and combinations of these operations.

Also included: Gr. 7 AF 1.3

Add Integers

Words to Remember

Inverse property of addition: The sum of a number and its opposite is 0.

$$\begin{aligned} a + (-a) &= 0 \\ -a + a &= 0 \end{aligned}$$

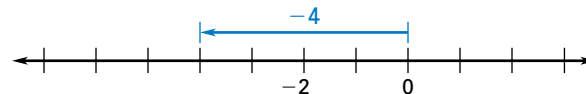
Getting Started You learned how to add integers using +1 and -1 cards. You can also use number lines and absolute values to add integers.

EXAMPLE 1 Adding Integers Using a Number Line

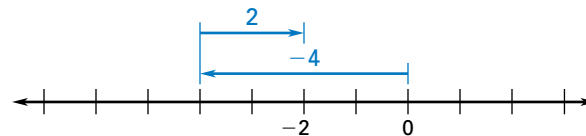
Use a number line to find the sum $-4 + 2$.

Solution

Step 1 Begin at 0 and draw an arrow left to 4 units to represent starting at -4 .



Step 2 Begin at -4 and draw an arrow right 2 units to represent adding 2.



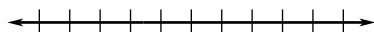
Step 3 Write the sum.

$-4 + 2 = -2$ since the end of the arrow after adding 2 to -4 is at -2 .

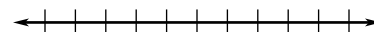
Adding Opposites When you add a number and its opposite on a number line, you draw an arrow away from zero (for example, +5 units) and then draw another arrow back in the opposite direction the same number of units (for example, -5 units). The result is 0.

TRY THIS Use a number line to find the sum.

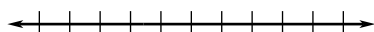
1. $3 + (-2) =$



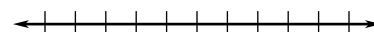
2. $-5 + 1 =$



3. $4 + (-4) =$



4. $-1 + (-2) =$



EXAMPLE 2**Adding Integers Using Absolute Values**

Use absolute values to find the sum $-6 + (-5)$.

Solution

Step 1 Find the absolute value of each term in the expression.

$$|-6| = 6 \text{ and } |-5| = 5$$

Step 2 Add the absolute values of the integers.

$$6 + 5 = 11$$

Step 3 Write the sum.

Since both of the original terms in the expression are negative, the sum must also be negative.

$$-6 + (-5)$$

$$\text{ANSWER } -6 + (-5) = -11$$

TRY THIS

Use absolute values to find the sum.

$$\begin{aligned} 5. \quad -7 + (-8) &= -(|-7| + |-8|) \\ &= -(\quad + \quad) \\ &= -(\quad) \\ &= \quad \end{aligned}$$

Adding Integers with Same Sign Use the sign of the integers in the original expression when using absolute values to add integers with like signs.

Adding Integers with Different Signs It is easiest to add integers with opposite signs using a number line, as shown in Example 1. To use absolute values to add integers that have opposite signs, subtract the least absolute value from the greater absolute value. Use the sign of the integer that has the greatest absolute value.

$$\begin{aligned} 3 + (-8) &= -(|-8| - |3|) \\ &= -(8 - 3) \\ &= -5 \end{aligned}$$

$|-8| > |3|$, so the final result will have the same sign as -8 .

Summarize

Using a Number Line to Add Integers

When an integer is negative, move that many units to the left of 0 or your starting point. When a number is positive, move that many units to the right of zero or your starting point.

Using Absolute Values to Add Integers

When the integers have the same sign, begin by finding the absolute values of the integers and then finding their sum. Then use the same sign as the sign of the original integers. When the integers have different signs, subtract the least absolute value from the greater absolute value. Use the sign of the integer with the greater absolute value.

Practice

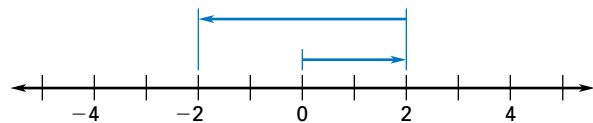
Match each sum with the number line that represents it.

1. $-4 + 2$ _____

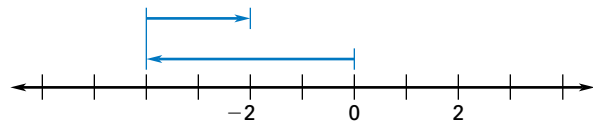
2. $2 + (-4)$ _____

3. $-2 + 4$ _____

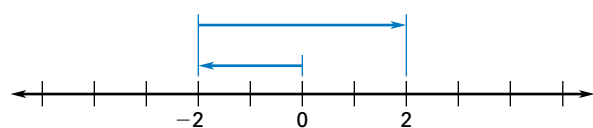
A.



B.

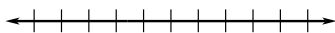


C.

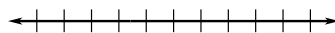


Find the sum using a number line.

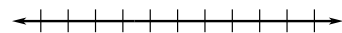
4. $3 + (-6) =$



5. $-7 + 6 =$



6. $7 + (-2) =$



Find the sum using absolute values.

7. $-4 + (-8) =$

8. $-12 + (-11) =$

9. $-9 + (-9) =$

10. $-15 + (-10) =$

11. $13 + (-6) + (-12) =$

12. $-70 + (-60) + (-28) =$

13. $-37 + (-4) + 18 =$

14. $17 + (-22) + 5 =$

Write a sum expression to represent the situation and state whether you would use a *number line* or *absolute values* to find the answer. Then solve the problem. Explain what your answer means.

- 15.** Yoko and Sheila are digging a hole for a fence post. The bottom of their hole is currently **10** inches below the surface of the ground. Yoko digs the hole **5** inches deeper and Sheila digs the hole **8** inches deeper. What is the elevation (in inches) of the bottom of the hole after Sheila is finished?

- 16.** Jasmine and Harley are planting flowers in their flower box. Jasmine fills a watering can with **3** gallons of water. Harley poured **1** gallon of water onto the flowers. How much water is in the watering can after Harley waters the flowers?



DID YOU GET IT?

- 17. Fill in the missing words.** To add two integers with opposite signs use a(n) _____. When a number in the sum is negative, then the arrow should move _____. When a number in the sum is positive, then the arrow should move _____.
- 18. Explain your reasoning.** When is it usually easiest to use absolute values to add integers?

LESSON

2-38



California Standards

Gr. 5 NS 2.1: Add, subtract, multiply, and divide with decimals; add with negative integers; **subtract positive integers from negative integers; and verify the reasonableness of the results.**

Gr. 6 NS 2.3: Solve addition, **subtraction**, multiplication, and division **problems**, including those arising in concrete situations, **that use positive and negative integers** and combinations of these operations.

Subtract Integers

Words to Remember

Opposites: Two numbers that are the same distance from **0** on a number line, but on opposite sides of **0**

3 and -3 are opposites.

8 and -8 are opposites.

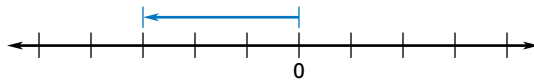
Subtraction Rule: To subtract an integer, add its opposite.

$$5 - 8 = 5 + (-8) = -3$$

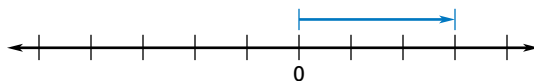
$$2 - (-3) = 2 + (3) = 5$$

Getting Started You added integers. Now you will subtract integers using a number line or the subtraction rule.

Using a Number Line When subtracting a *positive integer* using a number line, move to the left of the starting point.



When you are subtracting a *negative integer*, move instead to the right. Because you move to the left when you subtract a positive integer, you move in the opposite direction (to the right) when you subtract a negative integer.

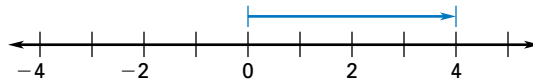


EXAMPLE 1 Subtracting a Positive Integer

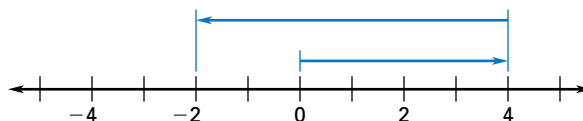
Find the difference $4 - 6$ using a number line.

Solution

Step 1 Move 4 units to the right of 0.



Step 2 Move 6 units to the left of 4.



The final position is -2 , so $4 - 6 = -2$.

Step 3 Use the subtraction rule of adding the opposite to check your answer.

$$4 - 6 = 4 + (-6) = -2$$

Remember

When subtracting a positive number, move to the left on the number line.

EXAMPLE 2**Subtracting a Negative Integer**

Use the rule for subtraction to find the difference $2 - (-3)$.

Solution

Step 1 Rewrite as adding the opposite.

$$2 - (-3) = 2 + (+3)$$

Step 2 Add.

$$2 + 3 = 5$$

Look Back

See Lesson 3-7 for help with adding integers.

EXAMPLE 3**Subtracting a Negative Integer**

Use the rule for subtraction to find the difference $-4 - (-2)$.

Solution

Step 1 Rewrite as adding the opposite.

$$-4 - (-2) = -4 + (+2)$$

Step 2 Add.

$$-4 + 2 = -2$$

EXAMPLE 4**Subtracting a Positive Integer**

Use the rule for subtraction to find the difference $-6 - 1$.

Solution

Step 1 Rewrite as adding the opposite.

$$-6 - 1 = -6 + (-1)$$

Step 2 Add.

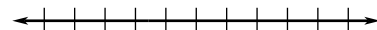
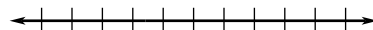
$$-6 + (-1) = -7$$

TRY THIS

Use a number line to find the difference.

1. $3 - 6 =$

2. $1 - 5 =$



Use the subtraction rule to find the difference.

3. $3 - (-5) =$

4. $-5 - 6 =$

Summarize**Subtracting Integers Using a Number Line**

To subtract a positive integer, move to the left. To subtract a negative integer, move to the right.

Subtracting Integers Using the Subtraction Rule

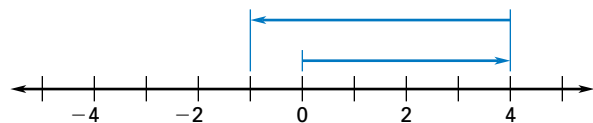
To subtract an integer, add its opposite.

Practice

Match each difference with the number line that represents it.

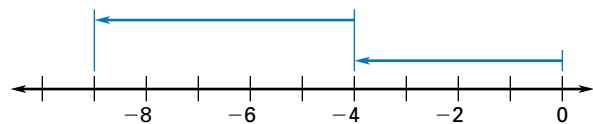
1. $-4 - 5$ _____

A.



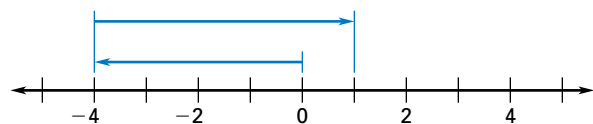
2. $-4 - (-5)$ _____

B.



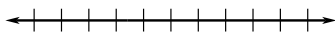
3. $4 - 5$ _____

C.

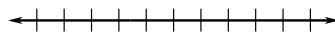


Find the difference using a number line.

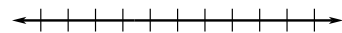
4. $3 - 7 =$



5. $-1 - 2 =$



6. $-2 - (-2) =$



Find the difference using the subtraction rule.

7. $-4 - (-8) =$

8. $2 - (-11) =$

9. $-9 - (-9) =$

10. $-15 - 10 =$

11. $3 - 10 =$

12. $12 - (-11) =$

13. $-13 - 13 =$

14. $-85 - (-20) =$

Write a difference expression to represent the situation. Then solve the problem and explain what your answer means.

- 15.** At noon the temperature was 84°F . Eight hours later the temperature had dropped 14°F . What was the temperature at 8:00 P.M.?

- 16.** Brandon is saving money to buy a bike that costs \$185. He currently has \$120 in his savings account. How much more does Brandon need to save so he can buy the bike?



- 17.** An elevator is stopped at a floor 20 feet below ground. It descends to another floor 65 feet below ground. What is the change in elevation of the elevator?

DID YOU GET IT?

- 18. Fill in the missing words.** To subtract a positive integer, move to the _____ on a number line. To subtract a negative integer, move to the _____ on a number line.

- 19. Describe a process.** Describe how to use the subtraction rule.

LESSON
2-39

California
Standards

Gr. 6 NS 2.3: Solve addition, subtraction, **multiplication**, and division **problems**, including those arising in concrete situations, **that use positive and negative integers** and combinations of these operations.

Gr. K/1/2 MR 1.1: Determine the **approach, materials, and strategies** to be used.

Also included: Gr. 3/4/5/6 MR 3.0

Multiply Integers

Words to Remember

Multiplication is the same as repeated addition.

$$4 \times 2 = 2 + 2 + 2 + 2 = 8$$

$$5(-6) = -6 + (-6) + (-6) + (-6) + (-6) = -30$$

Getting Started You added integers. Now you will multiply integers by using repeated addition. You will also learn some patterns about the sign of the product of two integers.

EXAMPLE 1

Multiplying Two Integers

Find the product by rewriting it as repeated addition.

- 3×15
- $5(-2)$
- $3 \times (-1)$

Solution

- $3 \times 15 = 15 + 15 + 15 = 45$
- $5(-2) = -2 + (-2) + (-2) + (-2) + (-2) = -10$
- $3 \times (-1) = -1 + (-1) + (-1) = -3$

TRY THIS

Use repeated addition to find the product.

1. 5×8

2. $2(-4)$

3. $4(6)$

4. $3 \times (-7)$

EXAMPLE 2**Finding a Pattern for Multiplication**

Look back at Example 1 and at the table below to find a pattern, or rule, for multiplying integers. What do you notice about the product of a positive integer and a negative integer? About the product of two negative integers?

| | |
|--------------------|----|
| $2 \times (-3) =$ | -6 |
| $1 \times (-3) =$ | -3 |
| $0 \times (-3) =$ | 0 |
| $-1 \times (-3) =$ | 3 |
| $-2 \times (-3) =$ | 6 |

Solution

The product of a positive integer and a negative integer is negative.

The product of two negative integers is positive.

TRY THIS

Will the product be *positive* or *negative*?

5. $46 \times (-8)$

6. $(-46) \times (-85)$

7. $(-46) \times (-85) \times (-85)$

8. $(-46) \times (-46) \times (-85) \times (-85)$

EXAMPLE 3**Multiplying Two Integers**

Use the rules for multiplying integers from Example 2 to find the product.

a. $4 \times (-4)$

b. $-5(-7)$

Solution

a. $4 \times (-4) = -16$

The product of a positive integer and a negative integer is negative.

b. $-5 \times (-7) = 35$

The product of two negative integers is positive.

TRY THIS

Use the multiplication rules to find the product.

9. $6(-10) =$

10. $-8 \times (-4) =$

Summarize

The product of two negative integers is positive.

The product of a positive integer and a negative integer is negative.

Practice

Match each product with the repeated addition that represents it.

1. $4 \times (-3)$ _____

A. $-3 + (-3) + (-3) + (-3)$

2. $3 \times (-4)$ _____

B. $4 + 4 + 4$

3. 3×4 _____

C. $-4 + (-4) + (-4)$

Find the product.

4. $10(5) =$

5. $4(-8) =$

6. $-12 \times 11 =$

7. $10 \times (-10) =$

8. $14 \times 3 =$

9. $-42 \times 0 =$

10. $8(-9) =$

11. $-20(-6) =$

12. $-33(1) =$

13. $2(-3)(-1) =$

14. $-3 \times 8 \times 2 =$

15. $12(-3)(5) =$

16. $-6 \times 4 \times 0 =$

17. $-3(2)(-4) =$

18. $-5(-7)(-1) =$

19. $12(2)(-8) =$

Write a product expression to represent the situation. Decide whether the numbers are positive or negative. Then simplify the expression and explain what the answer means.

- 20.** The temperature drops 2°F each hour. What is the total change in temperature after 12 hours?

- 21.** Sylvia burns 6 calories per minute when she runs. How many calories does she burn when she runs for 15 minutes?

- 22.** Chandler drives his car 20 miles round trip to work everyday. How many total miles does he drive to and from work in 5 days?



DID YOU GET IT?

- 23. Fill in the missing words.** The product of a negative integer and a positive integer is _____. The product of two negative integers is _____.

- 24. Explain your reasoning.** Use an example to help you explain how multiplication is like repeated addition.

LESSON 2-40



California Standards

Gr. 6 NS 2.3: Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Gr. 3/4/5/6/7 MR 2.0: Students use strategies, skills, and concepts in finding solutions.

Also included: Gr. 3/4/5 MR 3.3

Divide Integers

Words to Remember

Quotient: The result of dividing two integers

Related problem: You can rewrite a division problem as the related multiplication problem.

$$16 \div 8 = 2 \rightarrow 8 \times 2 = 16$$

$$2 \div 4 = 0.5$$

$$-5 \div 3 = -1.\bar{6}$$

quotient

Getting Started In Lesson 2-39 you multiplied integers. Now you will use multiplication and mental math to divide integers.

EXAMPLE 1

Rewriting Division as Multiplication

Rewrite the division problem as a related multiplication problem.

a. $48 \div (-6) = ?$

b. $-64 \div 8 = ?$

Solution

a. $48 \div (-6) = ?$ is related to the verbal multiplication expression:

“Some number times -6 is equal to 48 ,” so $? \times (-6) = 48$

b. $-64 \div 8 = ?$ is related to the verbal multiplication expression:

“Some number times 8 is equal to -64 ,” so $? \times 8 = -64$

Remember

One method for finding the quotient of two integers is to begin by rewriting the division problem as a multiplication problem.

TRY THIS

Rewrite the division problem as a related multiplication problem.

1. $36 \div (-3) = ?$

2. $-15 \div (-5) = ?$

Rules for the Division of Integers The rules for multiplication of integers also hold true for division. The quotient of two integers with the same sign is positive. The quotient of two integers with different signs is negative.

EXAMPLE 2 Dividing Integers

Determine whether the quotient will be positive or negative.

a. $75 \div (-15) = \underline{\quad ? \quad}$

b. $-36 \div (-6) = \underline{\quad ? \quad}$

Solution

- a. Since the integers in the expression have *different signs* (one positive and one negative), the quotient will be *negative*.
- b. Since the integers in the expression have the *same sign* (both are negative), the quotient will be *positive*.

EXAMPLE 3 Dividing Integers

Rewrite and solve a related multiplication equation for $-40 \div (-8) = \underline{\quad ? \quad}$.

Solution

Step 1 Think “What number times -8 equals -40 ?”

$$\boxed{} \times (-8) = -40$$

Step 2 Solve using mental math.

Since the integers -40 and -8 have the same sign, their quotient will be positive.

$$\boxed{5} \times (-8) = -40, \text{ so } -40 \div (-8) = 5$$

TRY THIS Find the quotient.

3. $88 \div 11 = \boxed{}$

4. $-32 \div 4 = \boxed{}$

5. $-125 \div -5 = \boxed{}$

6. $180 \div (-15) = \boxed{}$

Summarize**Finding the Quotient of Integers**

The quotient of two integers with the same sign is positive. The quotient of two integers with different signs is negative.

One method for finding the quotient of two integers is to begin by rewriting the division problem as a multiplication problem.

Practice

Tell if the quotient is *positive* (P) or *negative* (N).

1. $15 \div (-3)$ _____

2. $-144 \div (-12)$ _____

3. $-180 \div 18$ _____

4. $155 \div 5$ _____

Rewrite the quotient as a related multiplication problem.

5. $105 \div (-5) = ?$

6. $156 \div (-12) = ?$

7. $-99 \div 11 = ?$

8. $-32 \div (-4) = ?$

Find the quotient.

9. $-5 \div (-1) =$

10. $14 \div (-2) =$

11. $75 \div (-3) =$

12. $-64 \div (-8) =$

13. $21 \div 3 =$

14. $-60 \div (-5) =$

15. $36 \div 4 =$

16. $-36 \div 6 =$

Write a quotient expression to represent the situation. Then solve the problem. Explain what your answer means.

- 17.** Manuel has some strawberries that he is going to divide equally among himself and 4 friends. Suppose there are 75 strawberries. How many strawberries will each person get?

- 18.** A family of 3 shares the minutes on a cellular telephone equally. Suppose the cellular phone has 675 minutes per month. How many minutes does each person get to use each month?



DID YOU GET IT?

- 19. Fill in the missing words.** The quotient of two integers with the same sign will be a _____ number. The quotient of two integers with different signs will be a _____ number.
- 20. Compare.** How is dividing integers similar to multiplying integers?

LESSON
2-41**California Standards**

Gr. 6 NS 2.0: Students calculate and solve problems involving addition, subtraction, multiplication, and division.

Gr. 6 NS 2.3: Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Also included: Gr. 1/2 MR 2.0

Solve Problems with Integers

Strategies to Remember

When you solve a word problem, explain your reasoning:

Describe your thinking.

Show your work.

Getting Started When you explain your reasoning in solving a word problem, you can use steps to give a description of the process used to solve the problem. Often, the final step explains why the solution is a reasonable answer.

EXAMPLE 1

Solving a Division Problem

You are filling balloons with helium for your friend's birthday party. You have 54 feet of ribbon to tie to the balloons. It takes 3 feet for each balloon. How many balloons can you tie with the 54 feet of ribbon? Explain your reasoning.



Solution

Step 1 Identify the operation needed.

To find how many equal lengths of 3 feet can be taken from 54 feet, you need to divide.

Step 2 Divide 54 feet by 3 feet to find the number of balloons you can tie.

$$54 \div 3 = 18$$

Step 3 Check that 18 balloons is a reasonable answer.

Because 20 is slightly more than 18 and $20 \text{ balloons} \times 3 \text{ feet} = 60$ total feet, the total number of feet should be slightly less than 60 feet. Because 54 feet is slightly less than 60 feet, 18 balloons is reasonable.

ANSWER You can use the 54 feet of ribbon to tie ribbons on 18 balloons.

TRY THIS

Solve the problem. Explain your reasoning.

1. A roll of paper towels is 140 feet long. The roll needs to be divided into 4 equal sections for a project. How long will each section be?

Remember

See Exercise 25 on page 23 for a discussion of golf scores.

EXAMPLE 2**Solving a Subtraction Problem**

Kiesha has a golf score of -2 after the fourth hole. She decreases her score by 1 more after the sixth hole. What is Kiesha's score after the sixth hole?

Solution

Step 1 Identify the operation needed.

The word "decreases" lets you know that you need to use subtraction.

Step 2 Subtract. $-2 - 1 = -3$

Step 3 Check that a score of -3 is reasonable.

Because she is decreasing her score by 1, her score should be 1 unit to the left of -2 on a number line. -3 is 1 unit to the left of -2 on a number line.

ANSWER Kiesha's score after the sixth hole is -3

EXAMPLE 3**Solving a Problem Using Multiple Operations**

Kaleb has 20 cups of flour. He uses 8 cups to make banana bread. He wants to use the remaining flour to make zucchini bread. Each loaf of zucchini bread uses 3 cups of flour. How many loaves can he make?

Solution

Step 1 Identify the operations needed.

Subtract to find the number of cups of flour left after Kaleb makes the banana bread. Then divide by the number of cups needed for each loaf of zucchini bread.

Step 2 Subtract. $20 - 8 = 12$ cups

Step 3 Divide. $12 \div 3 = 4$ loaves

Step 4 Check that 4 loaves is a reasonable answer.

Because $4 \times 3 = 12$, 4 loaves is reasonable.

ANSWER Kaleb can make 4 loaves of zucchini bread.

TRY THIS

Solve the problem. Explain your reasoning.

2. The high temperature increases by 3°F each day for 5 days. On Day 1 the high temperature was 67°F . What was the high temperature on Day 6?

Summarize**Solving Word Problems**

- (1) Identify the operation or operations needed.
- (2) Perform the calculations and solve the problem.
- (3) Explain your thinking at each step.
- (4) Check your answer for reasonableness.

Practice

Match the situation with the operation needed to solve the problem.

- | | |
|--|--------------------------|
| 1. Lyle earned \$5 less than Kay. ____ | A. Addition |
| 2. 3 curtains each require 4 yards of fabric. ____ | B. Subtraction |
| 3. A 50 yard rope is split into 8 sections. ____ | C. Multiplication |
| 4. Kelli is 4 inches taller than Kim. ____ | D. Division |

Identify the operation suggested by the phrase.

- | | |
|-----------------------|------------------------------|
| 5. Twice as many cats | 6. 12 more fish |
| 7. 20 fewer flowers | 8. One-third as many minutes |

Fill in the missing information to solve the problem.

9. The high temperature on a Tuesday in winter is -15°F . The high temperature increases by 8°F on Wednesday. What is the high temperature on Wednesday?

Step 1 The word “increases” tells you that you need to _____.

Step 2 Calculate $-15 \bigcirc 8 = \square$

Step 3 The high temperature on Wednesday is \square .

10. Thirteen students each have 4 pets. How many pets do the 13 students have all together?

Step 1 Since groups are joined together, you need to _____.

Step 2 Calculate $13 \bigcirc 4 = \square$.

Step 3 The students have a total of \square pets.

Look back at Exercises 2 and 3 on page 170.

11. Explain how to find the number of yards of fabric needed for the curtains.
12. Find the length of rope in each section. Explain your reasoning.

Tell which operation(s) must be used to solve the problem. Then solve the problem. Explain your answer.

13. The eighth grade class is taking a trip to a science museum. There are 75 eighth graders and the school requires that there be 1 chaperone for every 15 students. How many chaperones are needed for the trip?

14. Simon has 3 times as many baseball cards as Trevor. Trevor has 15 baseball cards. Garret has 10 more baseball cards than Simon. How many baseball cards does Garret have?



DID YOU GET IT?

15. **Write a word problem.** Write a word problem using the integers below. Then complete the calculations to solve the problem. Explain how you decided what operations to use.

$$35 \text{ } \bullet \text{ } 7 = \text{ } \square$$

16. **Explain your reasoning.** Sometimes a word problem does not contain a key word or phrase that suggests a specific operation. How can you determine what operation should be used?

LESSON

2-42



California Standards

Gr. 7 NS 1.2: Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Gr. 3/4/5/6/7 MR 3.2: Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

Rational Number Operations

Words to Remember

Rational number: A number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

$$0.75 = \frac{3}{4}$$

$$2 = \frac{2}{1}$$

So, a rational number is any number that can be represented by a fraction.

$$-56 = \frac{-56}{1}$$

$$-0.\overline{6} = -\frac{2}{3}$$

Getting Started Previously you learned what rational numbers were. Now you are going to perform operations with rational numbers.

The rules you use to add, subtract, multiply, and divide integers also apply to rational numbers.

EXAMPLE 1

Adding and Subtracting Rational Numbers

Find the sum or difference.

a. $-\frac{3}{8} + \frac{3}{4}$

b. $2.3 - 5.22$

Solution

$$\begin{aligned} \text{a. } -\frac{3}{8} + \frac{3}{4} &= -\frac{3}{8} + \frac{6}{8} \\ &= \frac{-3+6}{8} \\ &= \frac{3}{8} \end{aligned}$$

Rewrite using LCD.

Add numerators.

Simplify.

b. $2.3 - 5.22 = 2.3 + (-5.22)$

To subtract a number, add its opposite.

$$= -(|5.22| - |2.3|)$$

Use rule for adding numbers with different signs.

$$= -2.92$$

Simplify.

TRY THIS

Find the sum or difference.

1. $-1\frac{1}{10} - \frac{4}{5} =$

2. $-1.125 + 3.721 =$

EXAMPLE 2**Dividing Rational Numbers**Find the quotient $\frac{2}{3} \div -\frac{1}{10}$.**Solution**

$$\frac{2}{3} \div \left(-\frac{1}{10}\right) = \frac{2}{3} \times \left(-\frac{10}{1}\right)$$

Multiply by reciprocal of $-\frac{1}{10}$.

$$= -\frac{20}{3}$$

Multiply. The numbers have different signs, so their product is negative.

$$= -6\frac{2}{3}$$

Simplify.

EXAMPLE 3**Multiplying Rational Numbers**Find the product -8.01×4.84 .**Solution**

$$-8.01 \times 4.84 = -38.7684$$

Multiply. The numbers have different signs, so their product is negative.

TRY THIS

Find the product or quotient.

3. $\frac{3}{4} \div \left(-\frac{1}{16}\right) = \boxed{}$

4. $-\frac{3}{5} \div 9 = \boxed{}$

5. $1\frac{5}{9} \div 4\frac{3}{5} = \boxed{}$

6. $(-7.92) \times (-1.25) = \boxed{}$

Remember

Just as in Lesson 3-3 when you compared rational numbers, it is easiest to add and subtract rational numbers if they are written in the same form, such as all decimals or all fractions.

EXAMPLE 4**Using Rational Numbers**Find the product and quotient of -2.5 and $\frac{1}{5}$.**Solution**

To find the product or quotient, write the numbers in the same form.

$$\begin{aligned} \text{Multiply: } -2.5 \times \frac{1}{5} &= -2.5 \times 0.2 \\ &= -0.50 \end{aligned}$$

Write $\frac{1}{5}$ as 0.2.

Multiply.

$$\begin{aligned} \text{Divide: } -2.5 \div \frac{1}{5} &= -2.5 \div 0.2 \\ &= -12.5 \end{aligned}$$

Write $\frac{1}{5}$ as 0.2.

Divide.

Summarize

A *rational number* is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Performing Operations with Rational Numbers

Rewrite the rational numbers so they are all in the same form, all decimals or all fractions. Then use the rules for performing operations with decimals or fractions.

Practice

Find the sum, difference, product, or quotient.

1. $2.4 - 0.4 =$

2. $-4\frac{4}{7} + 6\frac{3}{5} =$

3. $7 \times 6.8 =$

4. $-10.4 + 2.7 =$

5. $-\frac{1}{8} \div \frac{3}{4} =$

6. $-\frac{3}{7} - \frac{3}{7} =$

7. $98 \times \frac{5}{6} =$

8. $-\frac{12}{19} - \frac{1}{38} =$

9. $4.6 + 8.01 =$

10. $-\frac{7}{8} \div 2 =$

11. $6\frac{1}{6} + 3\frac{5}{12} =$

12. $-10.22 - 4.08 =$

13. $56.701 - (-23.99) =$

14. $3\frac{13}{15} \times \left(-2\frac{1}{3}\right) =$

15. $-14\frac{3}{4} \div 3 =$

16. $-3.24 + 0.625 + (-2.3) =$

17. $4.42 \times 5.1 \times 0.5 =$

18. $3.3 + 6.7 - 0.3 =$

Tell what operation you need to perform to solve the problem. Then write and solve an expression that represents the situation.

- 19.** You and a friend order a pizza. You eat $\frac{3}{10}$ of the pizza and your friend eats $\frac{2}{5}$ of the pizza. What fraction of the pizza did you and your friend eat altogether?

- 20.** Sigourney has a notebook with **150** sheets of colored paper in it. Suppose she uses **0.2** of the sheets. How many sheets does she use?

- 21.** Brady has a box of crackers that weighs **16.8** ounces. Suppose he is going to share the box equally among himself and three friends. How many ounces of crackers will each person have?



DID YOU GET IT?

- 22. Fill in the missing words.** A rational number is any number that can be written in the form _____.

- 23. Explain your reasoning.** How can you perform operations on rational numbers?

LESSON
2-43

California
Standards

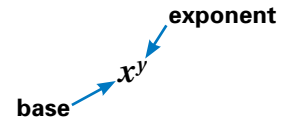
Gr. 7 NS 1.2: Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole number powers.**

Take Rational Numbers to Whole-Number Powers

Words to Remember

In a power the *base* is the repeated factor and the *exponent* is the number of times the factor is repeated.

For example, 2^5 means that you are going to multiply 5 factors of 2, or $2 \times 2 \times 2 \times 2 \times 2$.



Getting Started Previously you learned how to perform operations on rational numbers. Now you are going to learn how to raise a rational number to a power.

EXAMPLE 1
Taking an Integer to a Power

Evaluate the powers.

a. 4^3

b. 3^4

c. 5^2

Solution

Step 1

$$4^3 = 4 \times 4 \times 4$$

Write in expanded form.

$$3^4 = 3 \times 3 \times 3 \times 3$$

$$5^2 = 5 \times 5$$

Step 2

$$4^3 = 4 \times 4 \times 4 = 64$$

Multiply.

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$5^2 = 5 \times 5 = 25$$

TRY THIS

Evaluate the power.

1. $5^3 =$

2. $10^4 =$

EXAMPLE 2**Raising a Decimal to a Power**

Evaluate the powers.

a. $(4.1)^2$

b. $(2.2)^4$

Solution**Step 1** Write in expanded form.

$$(4.1)^2 = 4.1 \times 4.1$$

$$(2.2)^4 = 2.2 \times 2.2 \times 2.2 \times 2.2$$

Step 2 Multiply.

$$(4.1)^2 = 4.1 \times 4.1 = 16.81$$

$$(2.2)^4 = 2.2 \times 2.2 \times 2.2 \times 2.2 = 23.4256$$

TRY THIS

Evaluate the power.

3. $(1.8)^2 =$ _____

4. $(2.3)^3 =$ _____

EXAMPLE 3**Raising a Fraction to a Power**Evaluate the power $\left(\frac{2}{3}\right)^3$.**Solution**

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$= \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3}$$

$$= \frac{8}{27}$$

Write in expanded form.

Multiply numerators and multiply denominators.

Simplify.

Remember

When a fraction is raised to a power, both the numerator and the denominator are raised to that power.

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$

TRY THIS

Evaluate the power.

5. $\left(\frac{1}{4}\right)^3 =$ _____

6. $\left(\frac{4}{5}\right)^2 =$ _____

Summarize**Raising a Rational Number to a Power**

When you raise a rational number such as an integer, decimal, or fraction to a power, the exponent tells you how many times to use the base as a factor.

When you raise a fraction to a power, remember that both the numerator and the denominator are raised to that power.

Practice

Match the expression and its expanded form.

1. $(5.67)^2$ _____

A. $5.67 + 5.67$

B. 5.67×2

C. 5.67×5.67

D. $5.67 + 2$

2. $\left(\frac{1}{8}\right)^3$ _____

A. $\frac{1}{8} \times \frac{1}{8} \times \frac{1}{8}$

B. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

C. $\frac{1}{8} + 3$

D. $\frac{1}{8} \times 3$

Evaluate the power.

3. $2^4 =$

4. $\left(\frac{2}{5}\right)^2 =$

5. $(5.1)^3 =$

6. $(7.2)^2 =$

7. $12^2 =$

8. $\left(\frac{3}{4}\right)^4 =$

9. $\left(\frac{1}{2}\right)^5 =$

10. $(8.5)^3 =$

11. $13^3 =$

12. $(4.8)^2 =$

13. $5^6 =$

14. $\left(\frac{6}{7}\right)^4 =$

Write a power to represent the problem. Then evaluate the power.

- 15.** The area of a square is found by squaring the length of one of the sides. Suppose a side of a square patio has a length of **12.3** feet. What is the area of the top of the patio?

- 16.** A message about softball practice needs to get to members of all of the teams. The coach calls **3** teammates in the morning. Later, each of those **3** people calls **3** more teammates. Finally, each of those teammates calls **3** more people. How many teammates were called in the final, last round of calling?



- 17.** Tori is making a fruit pizza for her mother's birthday. She is halving the recipe, which means that she is making a pizza that requires half of each amount of the ingredients listed. Suppose $\frac{1}{2}$ cup of kiwifruit is needed in the original recipe. How much kiwifruit is needed for Tori's fruit pizza?

DID YOU GET IT?

- 18. Fill in the missing words.** To take a rational number to a power, the _____ tells you how many factors of the _____ to _____ together.
- 19. Explain your reasoning.** How do you take a fraction to a power?

Mixed Practice for Lessons 2-37 to 2-43

Vocabulary Review

Match the word with its mathematical meaning and its everyday meaning.

| Word | Mathematical meaning | Everyday meaning |
|-------------------------------|---|-----------------------------------|
| 1. add _____, _____ | A. the operation performed to find a difference | X. somebody that supports a cause |
| 2. subtract _____, _____ | B. determines how many times a number is a factor | Y. to withdraw or take away |
| 3. exponent _____, _____ | C. the operation performed to find a sum | Z. to say more about something |

Fill in the words.

4. When multiplying or dividing two integers, when both integers are negative the result is _____. When one integer is negative and one is positive, then the result is _____.

Add, subtract, multiply, or divide.

5. $-4 + 2 =$

6. $3 \times (-8) =$

7. $-18 \div (-2) =$

8. $5 - (-7) =$

9. $-12 + 19 =$

10. $-5 - (-20) =$

11. $50 \div (-25) =$

12. $-7 \times (-12) =$

13. $-99 \div 11 =$

14. $16 + (-17) =$

15. $-13 \times 4 =$

16. $-18 - 12 =$

17. Fill in the missing information.

At a softball game $\frac{5}{8}$ of the spectators are fans of the home team. $\frac{1}{5}$ of these fans leave before the end of the game. How many of the fans of the home team, expressed as a fraction of the total spectators, stayed for the entire game?

Step 1 The operations needed are _____ and _____.

Step 2 Calculate $1 \text{ } \bigcirc \text{ } \frac{1}{5} = \text{ } \square \text{ }$ of the home team fans stayed.

Step 3 Calculate $\frac{5}{8} \text{ } \bigcirc \text{ } \frac{4}{5} = \text{ } \square \text{ }$

Step 4 The fraction of the home team fans that stayed for the entire game is \square of the spectators.

Simplify the power.

18. $5^3 = \square$

19. $(0.3)^2 = \square$

20. $\left(\frac{3}{4}\right)^4 = \square$

21. $(1.2)^2 = \square$

Tell what operation(s) you need to find the answer. Then solve the problem. Explain your reasoning.

- 22.** Keith, Justin, and Jamal rent a movie. The total cost is **\$4.02**. If they are going to split the cost equally, how much does each person have to pay?

- 23.** Kara, Emma, and Julia are competing in a high school swim meet. Kara finishes the 100 yard freestyle event in $\frac{19}{20}$ minute, Emma finishes the event in 0.9 minute, and Julia finishes in $\frac{11}{12}$ minute. Who won the event?

